§1.1: HOW DO WE MEASURE VELOCITY?

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Lectures 1–2

Definition

For an object moving in a straight line with position function $s(t)$, the average velocity of the object from $t = a$ to $t = b$ is given by the formula

$$AV_{[a,b]} = \frac{s(b) - s(a)}{b - a}.$$

ACTIVITY 1.1.2 (DESMOS)

The following questions concern the position function given by $s(t) = 64 - 16(t-1)^2$

(a) Compute the average velocity of the ball on each of the following time intervals: $[0.4, 0.8]$, $[0.7, 0.8]$, $[0.79, 0.8]$, $[0.799, 0.8]$, $[0.8, 1.2]$, $[0.8, 0.9]$, $[0.8, 0.81]$, $[0.8, 0.801]$. Include units for each value.

(b) On the provided graph in Figure 1, sketch the line that passes through the points $A = (0.4, s(0.4))$ and $B = (0.8, s(0.8))$. What is the meaning of the slope of this line? In light of this meaning, what is a geometric way to interpret each of the values computed in the preceding question?

(c) Use a graphing utility to plot the graph of $s(t) = 64 - 16(t-1)^2$ on an interval containing the value $t = 0.8$. Then, zoom in repeatedly on the point $(0.8, s(0.8))$. What do you observe about how the graph appears as you view it more and more closely?

(d) What do you conjecture is the velocity of the ball at the instant $t = 0.8$? Why?

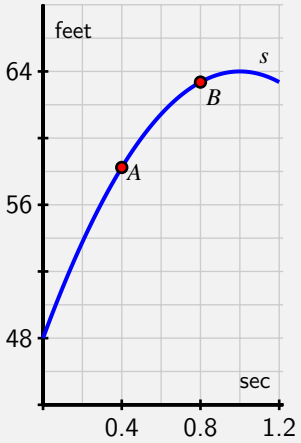

Figure 1: A partial plot of $s(t) = 64 - 16(t-1)^2$.

ACTIVITY 1.1.3 (DESMOS)

(a) Compute the average velocity of the ball on the time interval $[1.5, 2]$. What is different between this value and the average velocity on the interval $[0, 0.5]$?

(b) Use appropriate computing technology to estimate the instantaneous velocity of the ball at $t = 1.5$. Likewise, estimate the instantaneous velocity of the ball at $t = 2$. Which value is greater?

(c) How is the sign of the instantaneous velocity of the ball related to its behavior at a given point in time? That is, what does positive instantaneous velocity tell you the ball is doing? Negative instantaneous velocity?

(d) Without doing any computations, what do you expect to be the instantaneous velocity of the ball at $t = 1$? Why?

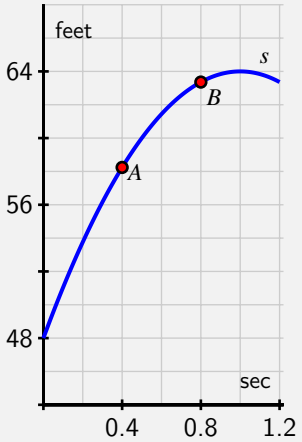

Figure 2: A partial plot of $s(t) = 64 - 16(t - 1)^2$.

ACTIVITY 1.1.4

For the function given by $s(t) = 64 - 16(t-1)^2$, find the most simplified expression you can for the average velocity of the ball on the interval $[2, 2+h]$. Use your result to compute the average velocity on $[1.5, 2]$ and to estimate the instantaneous velocity at $t = 2$. Finally, compare your earlier work in Activity 1.1.2.

§1.2: The Notion of Limit

What is a limit, and why should we care?

Preview Activity

Definition

Given a function $f(x)$, a fixed input $x = a$, and a real number L, we say that f has limit L as x approaches a, and write

$$\lim_{x \to a} f(x) = L$$

provided we can make $f(x)$ as close to L as we like by making x sufficiently close (but not equal) to a. If we cannot do this for any real number L, we say that f does not have a limit as $x \to a$.

Question

How can this definition fail?

Desmos

TWO WAYS OF CALCULATING: AN EXAMPLE

Let's calculate

$$\lim_{x \to 1} \frac{x^3 - 1}{x - 1}.$$

ACTIVITY 1.2.2 (DESMOS)

(a) $\lim_{x \to 1} \dfrac{x^2 - 1}{x - 1} = 2$

(b) $\lim_{x \to 0} \dfrac{(2 + x)^3 - 8}{x} = 12$

(c) $\lim_{x \to 0} \dfrac{\sqrt{x + 1} - 1}{x} = \dfrac{1}{12}$

Definition

Suppose an object moves with position given by $s(t)$. Then the instantaneous velocity of the object at $t = a$ is

$$IV_{t=a} := \lim_{h \to 0} \left(AV_{[a,a+h]} \right) = \lim_{h \to 0} \frac{s(a+h) - s(a)}{h}.$$

Consider a moving object whose position function is given by $s(t) = t^2$, where s is measured in meters and t is measured in minutes.

(a) Determine the most simplified expression for the average velocity of the object on the interval $[3, 3 + h]$, where $h > 0$.

(b) Determine the average velocity of the object on the interval $[3, 3.2]$. Include units on your answer.

(c) Determine the instantaneous velocity of the object when $t = 3$. Include units on your answer.

ACTIVITY 1.2.4

For the moving object whose position s at time t is given by the graph below, answer each of the following questions. Assume that s is measured in feet and t is measured in seconds.

(a) Use the graph to estimate the average velocity of the object on each of the following intervals: $[0.5, 1]$, $[1.5, 2.5]$, $[0, 5]$. Draw each line whose slope represents the average velocity you seek.

(b) How could you use average velocities or slopes of lines to estimate the instantaneous velocity of the object at a fixed time?

(c) Use the graph to estimate the instantaneous velocity of the object when $t = 2$. Should this instantaneous velocity at $t = 2$ be greater or less than the average velocity on $[1.5, 2.5]$ that you computed in (a)? Why

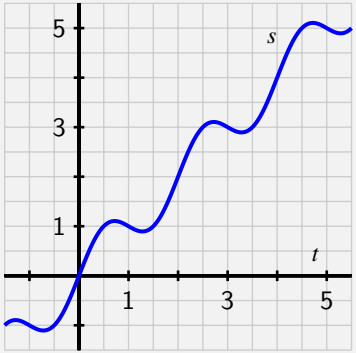

Figure 3: Plot of the position function $y = s(t)$ in Activity 1.2.4.