§1.3: THE DERIVATIVE AT A POINT

Dr. Janssen

Lecture 3

What is a derivative?

Preview Activity

Definition

Let $f(x)$ be a function and $x = a$ a value in the function's domain. We define the derivative of f with respect to x evaluated at $x = a$, denoted by $f'(a)$, by the formula

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h},$$

provided the limit exists. We say that f is differentiable at $x = a$.

THE ROAD

Chapter 1 is all about *understanding* derivatives.

Chapter 2 is all about *calculating* derivatives.

Chapter 3 is about *applying* derivatives.

Desmos Example 1.3

DERIVATIVES ARE SLOPES!!!!!!

EXAMPLE

Let's use algebra to find $f'(2)$ given $f(x) = x^2 + 3x$.

ACTIVITY 1.3.2

Consider the function f whose formula is $f(x) = 3 - 2x$.

(a) What familiar type of function is f? What can you say about the slope of f at every value of x?

(b) Compute the average rate of change of f on the intervals $[1, 4]$, $[3, 7]$, and $[5, 5 + h]$; simplify each result as much as possible. What do you notice about these quantities?

(c) Use the limit definition of the derivative to compute the exact instantaneous rate of change of f with respect to x at the value $a = 1$. That is, compute $f'(1)$ using the limit definition. Show your work. Is your result surprising?

(d) Without doing any additional computations, what are the values of $f'(2)$, $f'(\pi)$, and $f'(-\sqrt{2})$? Why?

ACTIVITY 1.3.3 (DESMOS)

A water balloon is tossed vertically in the air from a window. The balloon's height in feet at time t in seconds after being launched is given by $s(t) = -16t^2 + 16t + 32$. Use this function to respond to each of the following questions.

(a) Sketch an accurate, labeled graph of s on the axes provided.

(b) Compute the average rate of change of s on the time interval $[1, 2]$. Include units on your answer and write one sentence to explain the meaning of the value you found.

(c) Use the limit definition to compute the instantaneous rate of change of s with respect to time, t, at the instant $a = 1$. Show your work using proper notation, include units on your answer, and write one sentence to explain the meaning of the value you found.

(d) On your graph in (a), sketch two lines: one whose slope represents the average rate of change of s on $[1, 2]$, the other whose slope represents the instantaneous rate of change of s at the instant $a = 1$. Label each line clearly.

(e) For what values of a do you expect $s'(a)$ to be positive? Why? Answer the same questions when "positive" is replaced by "negative" and "zero."

ACTIVITY 1.3.4 (DESMOS)

A rapidly growing city in Arizona has its population P at time t, where t is the number of decades after the year 2010, modeled by the formula $P(t) = 25000e^{t/5}$. Use this function to respond to the following questions.

(a) Sketch an accurate graph of P for $t = 0$ to $t = 5$ on the axes provided. Label the scale on the axes carefully.

(b) Compute the average rate of change of P between 2030 and 2050. Include units on your answer and write one sentence to explain the meaning (in everyday language) of the value you found.

(c) Use the limit definition to write an expression for the instantaneous rate of change of P with respect to time, t, at the instant $a = 2$. Explain why this limit is difficult to evaluate exactly.

(d) Estimate the limit in (c) for the instantaneous rate of change of P at the instant $a = 2$ by using several small h values. Once you have determined an accurate estimate of $P'(2)$, include units on your answer, and write one sentence (using everyday language) to explain the meaning of the value you found.

(e) On your graph above, sketch two lines: one whose slope represents the average rate of change of P on $[2, 4]$, the other whose slope represents the instantaneous rate of change of P at the instant $a = 2$.

(f) In a carefully-worded sentence, describe the behavior of $P'(a)$ as a increases in value. What does this reflect about the behavior of the given function P?