§1.4: THE DERIVATIVE FUNCTION

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Lecture 4

What is the derivative function?

Preview Activity

Definition

Let $f(x)$ be a function and x a value in the function's domain. We define the derivative of f with respect to x, denoted by $f'(x)$, by the formula

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h},$$

provided the limit exists.

Example (Desmos)

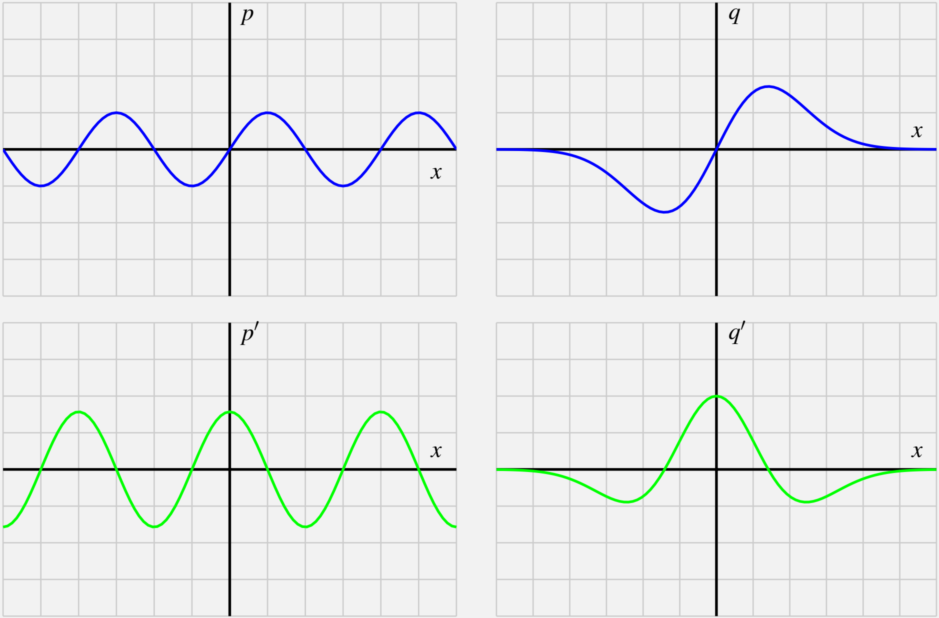
TWO PERSPECTIVES

- Given a graph of $y = f(x)$, how do we construct the graph of the derivative $y = f'(x)$?
- Given an algebraic definition for $y = f(x)$, how do we find an algebraic definition for $y = f'(x)$?

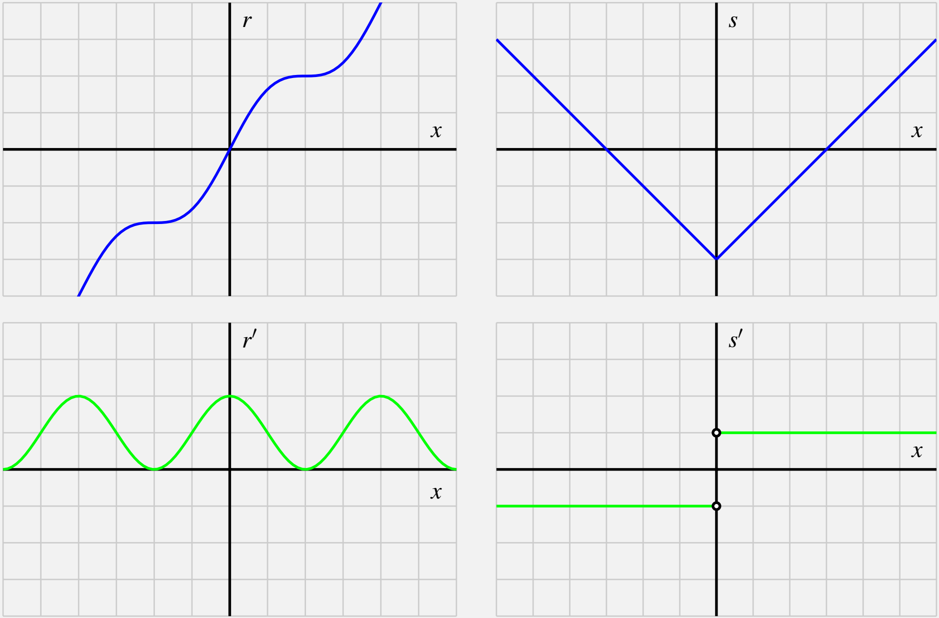
ACTIVITY 1.4.2

For each given graph of $y = f(x)$, sketch an approximate graph of its derivative function, $y = f'(x)$, on the axes immediately below. The scale of the grid for the graph of f is 1×1; assume the horizontal scale of the grid for the graph of f' is identical to that for f. If necessary, adjust and label the vertical scale on the axes for f'.

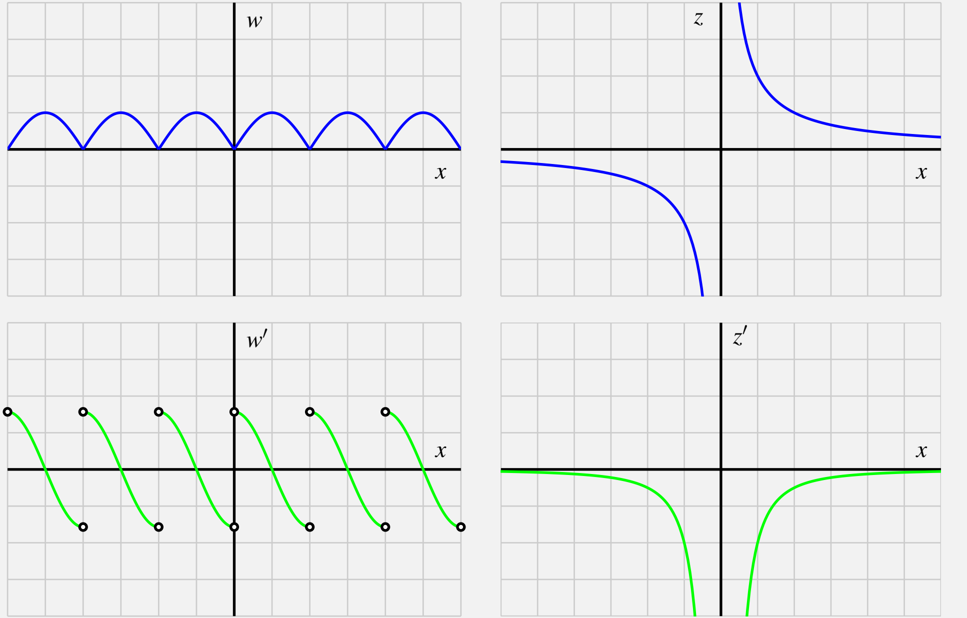
ACTIVITY 1.4.2



ACTIVITY 1.4.2



ACTIVITY 1.4.2



ACTIVITY 1.4.3

For each of the listed functions, determine a formula for the derivative function. For the first two, determine the formula for the derivative by thinking about the nature of the given function and its slope at various points; do not use the limit definition. For the latter four, use the limit definition.

(a) $f(x) = 1$ $\quad\Rightarrow\quad$ $f'(x) = 0$

(b) $g(t) = t$; $\quad\Rightarrow\quad$ $g'(t) = 1$

(c) $p(z) = z^2$; $\quad\Rightarrow\quad$ $p'(z) = 2z$

(d) $q(s) = s^3$; $\quad\Rightarrow\quad$ $q'(s) = 3s^2$

(e) $F(t) = \frac{1}{t}$; $\quad\Rightarrow\quad$ $F'(t) = -\frac{1}{t^2}$

(f) $G(y) = \sqrt{y}$; $\quad\Rightarrow\quad$ $G'(y) = \frac{1}{2\sqrt{y}}$

§1.5: Interpreting, Estimating, and Using the Derivative

How can we approximate and interpret derivatives of functions not defined by algebraic expressions?

GOAL: ACCURATE ESTIMATES OF THE DERIVATIVE

Let $f(x)$ be a function, a a point in the function's domain, and $h > 0$.

- The difference quotient is the expression $\dfrac{f(a+h) - f(a)}{h}$.
- The central difference is the expression $\dfrac{f(a+h) - f(a-h)}{2h}$.

Example (Desmos)

ACTIVITY 1.5.2

A potato is placed in an oven, and the potato's temperature F (in degrees Fahrenheit) at various points in time is taken and recorded in the following table. Time t is measured in minutes.

t	$F(t)$
0	70
15	180.5
30	251
45	296
60	324.5
75	342.8
90	354.5

(a) Use a central difference to estimate the instantaneous rate of change of the temperature of the potato at $t = 30$. Include units on your answer.

(b) Use a central difference to estimate the instantaneous rate of change of the temperature of the potato at $t = 60$. Include units on your answer.

(c) Without doing any calculation, which do you expect to be greater: $F'(75)$ or $F'(90)$? Why?

(d) Suppose it is given that $F(64) = 330.28$ and $F'(64) = 1.341$. What are the units on these two quantities? What do you expect the temperature of the potato to be when $t = 65$? when $t = 66$? Why?

(e) Write a couple of careful sentences that describe the behavior of the temperature of the potato on the time interval $[0, 90]$, as well as the behavior of the instantaneous rate of change of the temperature of the potato on the same time interval.

ACTIVITY 1.5.3

A company manufactures rope, and the total cost of producing r feet of rope is $C(r)$ dollars.

(a) What does it mean to say that $C(2000) = 800$?

(b) What are the units of $C'(r)$?

(c) Suppose that $C(2000) = 800$ and $C'(2000) = 0.35$. Estimate $C(2100)$, and justify your estimate by writing at least one sentence that explains your thinking.

(d) Which of the following statements do you think is true, and why?

- $C'(2000) < C'(3000)$
- $C'(2000) = C'(3000)$
- $C'(2000) > C'(3000)$

(e) Suppose someone claims that $C'(5000) = -0.1$. What would the practical meaning of this derivative value tell you about the approximate cost of the next foot of rope? Is this possible? Why or why not?

ACTIVITY 1.5.4

Researchers at a major car company have found a function that relates gasoline consumption to speed for a particular model of car. In particular, they have determined that the consumption C, in **liters per kilometer**, at a given speed s, is given by a function $C = f(s)$, where s is the car's speed in **kilometers per hour**.

(a) Data provided by the car company tells us that $f(80) = 0.015$, $f(90) = 0.02$, and $f(100) = 0.027$. Use this information to estimate the instantaneous rate of change of fuel consumption with respect to speed at $s = 90$. Be as accurate as possible, use proper notation, and include units on your answer.

(b) By writing a complete sentence, interpret the meaning (in the context of fuel consumption) of "$f(80) = 0.015$."

(c) Write at least one complete sentence that interprets the meaning of the value of $f'(90)$ that you estimated in (a).