§1.8: THE TANGENT LINE APPROXIMATION

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Lecture 8

How do we find the equation for a tangent line–and why do we care?

Preview Activity (Desmos)

Definition

Given a function $f(x)$ that is differentiable at $x = a$, the equation of the line tangent to $y = f(x)$ at $x = a$ is

$$y - f(a) = f'(a)(x - a)$$

The local linearization of $f(x)$ centered at $x = a$ is the linear function

$$L(x) = f'(a)(x - a) + f(a)$$

EXAMPLE 1.8.1 (DESMOS)

Let's find the local linearization of $f(x) = \sin(x)$ centered at $a = 0$, *given that* $f'(0) = 1$ (not obvious!).

ACTIVITY 1.8.2

Suppose it is known that for a given differentiable function $y = g(x)$, its local linearization at the point where $a = -1$ is given by $L(x) = -2 + 3(x + 1)$.

(a) Compute the values of $L(-1)$ and $L'(-1)$.

(b) What must be the values of $g(-1)$ and $g'(-1)$? Why?

(c) Do you expect the value of $g(-1.03)$ to be greater than or less than the value of $g(-1)$? Why?

(d) Use the local linearization to estimate the value of $g(-1.03)$.

(e) Suppose that you also know that $g''(-1) = 2$. What does this tell you about the graph of $y = g(x)$ at $a = -1$?

(f) For x near -1, sketch the graph of the local linearization $y = L(x)$ as well as a possible graph of $y = g(x)$ on the axes provided.

THE LINEARIZATION AND CONCAVITY

We know the linearization is an approximation, but is it an underestimate or an overestimate?

- When $f''(a) > 0$, $L(x)$ underestimates $f(x)$ for x near a
- When $f''(a) < 0$, $L(x)$ overestimates $f(x)$ for x near a
- When $f''(a) = 0$, we can't tell

ACTIVITY 1.8.3

This activity concerns a function $f(x)$ about which the following information is known:

- f is a differentiable function defined at every real number x
- $f(2) = -1$
- $y = f'(x)$ has its graph given in Figure 1

Your task is to determine as much information as possible about f (especially near the value $a = 2$) by responding to the questions below.

(a) Find a formula for the tangent line approximation, $L(x)$, to f at the point $(2, -1)$.

(b) Use the tangent line approximation to estimate the value of $f(2.07)$. Show your work carefully and clearly.

(c) Sketch a graph of $y = f''(x)$ on the righthand grid in Figure 1; label it appropriately.

(d) Is the slope of the tangent line to $y = f(x)$ increasing, decreasing, or neither when $x = 2$? Explain.

(e) Sketch a possible graph of $y = f(x)$ near $x = 2$ on the lefthand grid in Figure 1. Include a sketch of $y = L(x)$ (found in part (a)). Explain how you know the graph of $y = f(x)$ looks like you have drawn it.

(f) Does your estimate in (b) over- or under-estimate the true value of $f(2.07)$? Why?

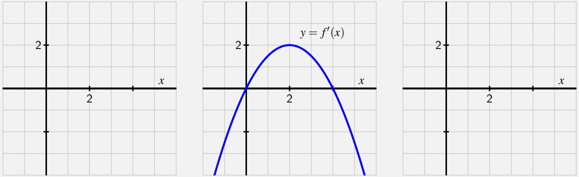

Figure 1: At center, a graph of $y = f'(x)$; at left, axes for plotting $y = f(x)$; at right, axes for plotting $y = f''(x)$.