§2.1-2.2: ELEMENTARY DERIVATIVE RULES/DERIVATIVES OF SINE AND COSINE

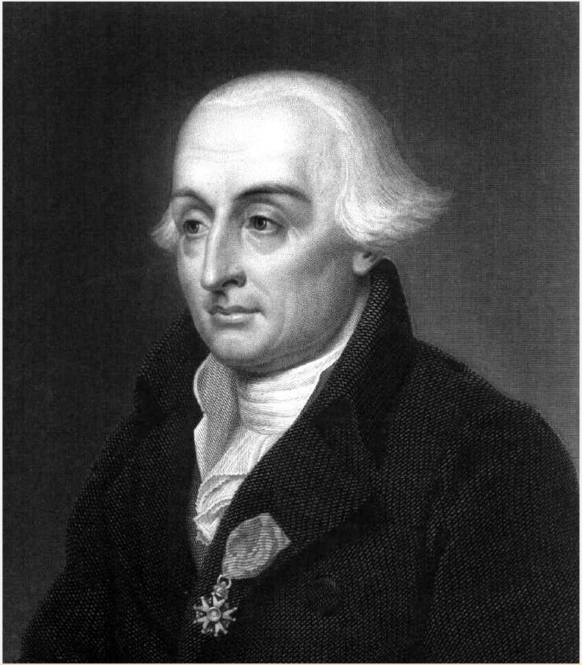
Dr. Janssen

Lecture 9

How can we quickly calculate derivatives of simple algebraic functions?

JOSEPH LOUIS-LAGRANGE (1736–1813)

- Italian/French mathematician
- Influential in analysis, number theory, and classical/celestial mechanics
- First to use prime notation



GOTTFRIED WILHELM LEIBNIZ (1646–1716)

- German philosopher and mathematician
- Co-inventor (with I. Newton) of calculus
- Notation: $\frac{dy}{dx}$

SOME DERIVATIVE RULES

Theorem

Let x be a variable, c and a constant, and $n \geq 1$ an integer. Then:

- $\dfrac{d}{dx}[c] = 0$
- $\dfrac{d}{dx}\left[x^n\right] = nx^{n-1}$
- $\dfrac{d}{dx}\left[a^x\right] = \ln(a)a^x.$

ACTIVITY 2.1.2

(a) $f(t) = \pi \Rightarrow f'(t) = 0$

(b) $g(z) = 7^z \Rightarrow g'(z) = \ln(7) \cdot 7^z$

(c) $h(w) = w^{3/4} \Rightarrow h'(w) = \frac{3}{4} w^{-1/4}$

(d) $p(x) = 3^{1/2} \Rightarrow p'(x) = 0$

(e) $r(t) = \left(\sqrt{2}\right)^t \Rightarrow r'(t) = \ln(\sqrt{2}) \left(\sqrt{2}\right)^t$

(f) $s(q) = q^{-1} \Rightarrow s'(q) = -q^{-2} = -\frac{1}{q^2}$

(g) $m(t) = \frac{1}{t^3} = t^{-3} \Rightarrow m'(t) = -3t^{-4}$

SUMS AND CONSTANT MULTIPLES

Theorem

Let $f(x)$ and $g(x)$ be differentiable and k a constant. Then:

- $\dfrac{d}{dx}[kf(x)] = kf'(x)$
- $\dfrac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$

ACTIVITY 2.1.3

(a) $f(x) = x^{5/3} - x^4 + 2^x \Rightarrow f'(x) = \frac{5}{3}x^{2/3} - 4x^3 + \ln(2)2^x$

(b) $g(x) = 14e^x + 3x^5 - x \Rightarrow g'(x) = 14e^x + 5 \cdot 3x^4 - 1$

(c) $h(z) = \sqrt{z} + \frac{1}{z^4} + 5^z \Rightarrow h'(z) = \frac{1}{2}z^{-1/2} - 4z^{-5} + \ln(5)5^z$

(d) $r(t) = \sqrt{53}t^7 - \pi e^t + e^4 \Rightarrow r'(t) = 7\sqrt{53}t^6 - \pi e^t$

(e) $s(y) = (y^2 + 1)(y^2 - 1) = y^4 - 1 \Rightarrow s'(y) = 4y^3$

(f) $q(x) = \frac{x^3 - x + 2}{x} = x^2 - 1 + 2x^{-1} \Rightarrow q'(x) = 2x - 2x^{-2}$

(g) $p(a) = 3a^4 - 2a^3 + 7a^2 - a + 12 \Rightarrow p'(a) = 12a^3 - 6a^2 + 14a - 1$

ACTIVITY 2.1.4

Each of the following questions asks you to use derivatives to answer key questions about functions. Be sure to think carefully about each question and to use proper notation in your responses.

(a) Find the slope of the tangent line to $h(z) = \sqrt{z} + \frac{1}{z}$ at the point where $z = 4$.

(b) A population of cells is growing in such a way that its total number in millions is given by the function $P(t) = 2(1.37)^t + 32$, where t is measured in days.

   i. Determine the instantaneous rate at which the population is growing on day 4, and include units on your answer.

   ii. Is the population growing at an increasing rate or growing at a decreasing rate on day 4? Explain.

(c) Find an equation for the tangent line to the curve $p(a) = 3a^4 - 2a^3 + 7a^2 - a + 12$ at the point where $a = -1$.

(d) What is the difference between being asked to find the *slope* of the tangent line (asked in (a)) and the *equation* of the tangent line (asked in (c))?