§2.3: PRODUCT AND QUOTIENT RULES

Dr. Janssen

Lecture 11

How do we calculate the derivatives of $f(x) \cdot g(x)$ and $\frac{f(x)}{g(x)}$?

Idea of the Product Rule

EXAMPLE

Let’s calculate $\frac{d}{dx}\left[x^2 \cdot e^x\right]$.

ACTIVITY 2.3.2

Use the product rule to answer each of the questions below. Throughout, be sure to carefully label any derivative you find by name. It is not necessary to algebraically simplify any of the derivatives you compute.

(a) Let $m(w) = 3w^{17}4^w$. Find $m'(w)$. $m'(w) = 3w^{17} \cdot 4^w \ln(4) + 4^w \cdot 3 \cdot 17w^{16}$

(b) Let $h(t) = (\sin(t) + \cos(t))t^4$. Find $h'(t)$.
$h'(t) = (\sin(t) + \cos(t)) \cdot 4t^3 + t^4 \cdot (\cos(t) - \sin(t))$

(c) Determine the slope of the tangent line to the curve $y = f(x)$ at the point where $a = 1$ if f is given by the rule $f(x) = e^x \sin(x)$.
$f'(1) = e(\cos(1) + \sin(1)) \approx 3.756$

(d) Find the tangent line approximation $L(x)$ to the function $y = g(x)$ at the point where $a = -1$ if g is given by the rule $g(x) = (x^2 + x)2^x$. $L(x) = -\frac{1}{2}(x + 1)$

The idea of the quotient rule

EXAMPLE

Let's calculate $\frac{d}{dx}\left[\frac{e^x}{x^2}\right]$.

ACTIVITY 2.3.3

Use the quotient rule to answer each of the questions below. Throughout, be sure to carefully label any derivative you find by name. That is, if you're given a formula for $f(x)$, clearly label the formula you find for $f'(x)$. It is not necessary to algebraically simplify any of the derivatives you compute.

(a) Let $r(z) = \frac{3^z}{z^4+1}$. Find $r'(z)$. $r'(z) = \dfrac{(z^4+1)3^z \ln(3) - 3^z(4z^3)}{(z^4+1)^2}$

(b) Let $v(t) = \frac{\sin(t)}{\cos(t)+t^2}$. Find $v'(t)$. $v'(t) = \dfrac{(\cos(t)+t^2)\cos(t) - \sin(t)(-\sin(t)+2t)}{(\cos(t)+t^2)^2}$

(c) Determine the slope of the tangent line to the curve $R(x) = \dfrac{x^2-2x-8}{x^2-9}$ at the point where $x = 0$. $R'(0) = 2/9$

(d) When a camera flashes, the intensity I of light seen by the eye is given by the function

$$I(t) = \frac{100t}{e^t},$$

where I is measured in candles and t is measured in milliseconds. Compute $I'(0.5)$, $I'(2)$, and $I'(5)$; include appropriate units on each value; and discuss the meaning of each. We find $I'(0.5) = \frac{50}{e^{0.5}} \approx 30.327$, $I'(2) = \frac{-100}{e^2} \approx -13.534$, and $I'(5) = \frac{-400}{e^4} \approx -2.695$, each measured in candles per millisecond.

Let's find $f'(x)$ given $f(x) = \dfrac{3x^2}{e^x \cos x + 2x}$.

ACTIVITY 2.3.4: PUTTING IT ALL TOGETHER

Use relevant derivative rules to answer each of the questions below. Throughout, be sure to use proper notation and carefully label any derivative you find by name.

(a) Let $f(r) = (5r^3 + \sin(r))(4^r - 2\cos(r))$. Find $f'(r)$.
$f'(r) = (5r^3 + \sin(r))[4^r \ln(4) + 2\sin(r)] + (4^r - 2\cos(r))[15r^2 + \cos(r)]$

(b) Let $p(t) = \dfrac{\cos(t)}{t^6 \cdot 6^t}$. Find $p'(t)$. $p'(t) = \dfrac{t^6 \cdot 6^t[-\sin(t)] - \cos(t)[t^6 \cdot 6^t \ln(6) + 6^t \cdot 6t^5]}{(t^6 \cdot 6^t)^2}$

(c) Let $g(z) = 3z^7 e^z - 2z^2 \sin(z) + \frac{z}{z^2+1}$. Find $g'(z)$.
$g'(z) = 3[z^7 e^z + 7z^6 e^z] - 2[z^2 \cos(z) + 2z\sin(z)] + \dfrac{(z^2+1)\cdot 1 - z(2z)}{(z^2+1)^2}$

(d) A moving particle has its position in feet at time t in seconds given by the function $s(t) = \frac{3\cos(t) - \sin(t)}{e^t}$. Find the particle's instantaneous velocity at the moment $t = 1$.
$s'(1) = \frac{-2\sin(1) - 4\cos(1)}{e^1} \approx -1.414$ ft/s

(e) Suppose that $f(x)$ and $g(x)$ are differentiable functions and it is known that $f(3) = -2$, $f'(3) = 7$, $g(3) = 4$, and $g'(3) = -1$. If $p(x) = f(x) \cdot g(x)$ and $q(x) = \dfrac{f(x)}{g(x)}$, calculate $p'(3)$ and $q'(3)$. $p'(3) = 30$ and $q'(3) = 13/8$