§2.4: DERIVATIVES OF OTHER TRIG FUNCTIONS

Dr. Janssen

Lecture 12

On Active Learning

PNAS METASTUDY


RESEARCH ARTICLE | PSYCHOLOGICAL AND COGNITIVE SCIENCES

Active learning increases student performance in science, engineering, and mathematics

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Edited* by Bruce Alberts, University of California, San Francisco, CA, and approved April 15, 2014 (received for review October 8, 2013)

May 12, 2014 | 111 (23) 8410-8415 | https://doi.org/10.1073/pnas.1319030111

VIEW RELATED CONTENT + THIS ARTICLE HAS A REPLY +

35,163 3,067

Highlights:

- Exam scores improved by an average of 6%
- Students 1.5 times as likely to fail in lecture-based classes
- “If the experiments analyzed here had been conducted as randomized controlled trials of medical interventions, they may have been stopped for benefit–meaning that enrolling patients in the control condition might be discontinued because the treatment being tested was clearly more beneficial.”

How do we calculate the derivatives of the other four trig functions?

PREVIEW ACTIVITY

Let's find the derivative of $f(x) = \tan(x)$.

$\cot(x)$

ACTIVITY 2.4.2

Let $h(x) = \sec(x)$ and recall that $\sec(x) = \frac{1}{\cos(x)}$.

(a) What is the domain of h? The domain of h is all real numbers x such that $x \neq \frac{k\pi}{2}$, where $k = \pm 1, \pm 2, \ldots$

(b) Use the quotient rule to develop a formula for $h'(x)$ that is expressed completely in terms of $\sin(x)$ and $\cos(x)$. $h'(x) = \frac{0 - 1(-\sin(x))}{\cos^2(x)} = \frac{\sin(x)}{\cos^2(x)}$

(c) How can you use other relationships among trigonometric functions to write $h'(x)$ only in terms of $\tan(x)$ and $\sec(x)$? $h'(x) = \sec(x)\tan(x)$

(d) What is the domain of h'? How does this compare to the domain of h? It's the same!

ACTIVITY 2.4.3

Let $p(x) = \csc(x)$ and recall that $\csc(x) = \frac{1}{\sin(x)}$.

(a) What is the domain of p? The domain of h is all real numbers x such that $xk\pi$, where $k = 0, \pm 1, \pm 2, \ldots$

(b) Use the quotient rule to develop a formula for $p'(x)$ that is expressed completely in terms of $\sin(x)$ and $\cos(x)$. $h'(x) = \frac{0-1\cdot(\cos(x))}{\sin^2(x)} = -\frac{\cos(x)}{\sin^2(x)}$

(c) How can you use other relationships among trigonometric functions to write $p'(x)$ only in terms of $\cot(x)$ and $\csc(x)$? $h'(x) = -\csc(x)\cot(x)$

(d) What is the domain of p'? How does this compare to the domain of p? It's the same!

SUMMARY

We have:

- $\frac{d}{dx}[\tan(x)] = \sec^2(x)$
- $\frac{d}{dx}[\cot(x)] = -\csc^2(x)$
- $\frac{d}{dx}[\sec(x)] = \sec(x)\tan(x)$
- $\frac{d}{dx}[\csc(x)] = -\csc(x)\cot(x)$

Answer each of the following questions. Where a derivative is requested, be sure to label the derivative function with its name using proper notation.

(a) Let $f(x) = 5\sec(x) - 2\csc(x)$. Find the slope of the tangent line to f at the point where $x = \frac{\pi}{3}$. $f'(\pi/3) = 10\sqrt{3} + \frac{4}{3}$

(b) Let $p(z) = z^2 \sec(z) - z\cot(z)$. Find the instantaneous rate of change of p at the point where $z = \frac{\pi}{4}$. $p'(\pi/4) = \frac{\pi^2}{16}\sqrt{2} + \frac{\sqrt{2}\pi}{2} + \frac{\pi}{2} - 1$

(c) Let $h(t) = \dfrac{\tan(t)}{t^2+1} - 2e^t\cos(t)$. Find $h'(t)$.

$h'(t) = \frac{(t^2+1)\sec^2(t) - 2t\tan(t)}{(t^2+1)^2} + 2e^t\sin(t) - 2e^t\cos(t)$

(d) Let $g(r) = \dfrac{r\sec(r)}{5^r}$. Find $g'(r)$. $g'(r) = \frac{r\sec(r)\tan(r) + \sec(r) - r5^r\sec(r)}{5^r}$

ACTIVITY 2.4.4E

When a mass hangs from a spring and is set in motion, the object's position oscillates in a way that the size of the oscillations decrease. This is usually called a *damped oscillation*. Suppose that for a particular object, its displacement from equilibrium (where the object sits at rest) is modeled by the function

$$s(t) = \frac{15\sin(t)}{e^t}.$$

Assume that s is measured in inches and t in seconds. Sketch a graph of this function for $t \geq 0$ to see how it represents the situation described. Then compute ds/dt, state the units on this function, and explain what it tells you about the object's motion. Finally, compute and interpret $s'(2)$.

By the quotient rule,

$$\frac{ds}{dt} = \frac{e^t \cdot 15\cos(t) - 15\sin(t)\cdot e^t}{(e^t)^2} = \frac{15\cos(t) - 15\sin(t)}{e^t}.$$

The function $\frac{ds}{dt} = s'(t)$ measures the instantaneous vertical velocity of the mass that is attached to the spring. In particular, $s'(2) = \frac{15\cos(2) - 15\sin(2)}{e^2} \approx -2.69$ inches per second, which tells us at the instant $t = 2$, the mass is moving downward at an instantaneous rate of 2.69 inches per second.