§2.5: THE CHAIN RULE

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Lecture 13

How do we calculate the derivative of a function formed by composition?

Preview Activity Discussion

Theorem

Let $g(x)$ be a function differentiable at a, and f a function differentiable at $g(a)$. Then $C = f(g(x))$ is differentiable at a, and

$$C'(a) = f'(g(a))g'(a).$$

EXAMPLE

Let $f(x) = (\tan x)^2$, and let's calculate $f'(x)$ in two different ways.

ACTIVITY 2.5.2

(a) $h(x) = \cos(x^4) \Rightarrow h'(x) = -\sin(x^4) \cdot 4x^3$

(b) $p(x) = \sqrt{\tan(x)} \Rightarrow p'(x) = \frac{1}{2}(\tan(x))^{-1/2}\sec^2(x)$

(c) $s(x) = 2^{\sin(x)} \Rightarrow s'(x) = \ln(2)2^{\sin(x)}\cos(x)$

(d) $z(x) = \cot^5(x) \Rightarrow z'(x) = 5\cot^4(x) \cdot (-\csc^2(x))$

(e) $m(x) = (\sec(x) + e^x)^9 \Rightarrow m'(x) = 9(\sec(x) + e^x)^8 \cdot (\sec(x)\tan(x) + e^x)$

USING MULTIPLE RULES: AN EXAMPLE

Let's calculate $f'(x)$ given

$$f(x) = \frac{e^{4x^2}}{\cos(\cos(x)) + x}$$

ACTIVITY 2.5.3

(a) $p(r) = 4\sqrt{r^6 + 2e^r} \Rightarrow p'(r) = 4 \cdot \frac{1}{2}(r^6 + 2e^r)^{-1/2} \cdot (6r^5 + 2e^r)$

(b) $m(v) = \sin(v^2)\cos(v^3) \Rightarrow m'(v) = \sin(v^2) \cdot (-\sin(v^3) \cdot (3v^2)) + \cos(v^3) \cdot \cos(v^2) \cdot (2v)$

(c) $h(y) = \dfrac{\cos(10y)}{e^{4y} + 1} \Rightarrow h'(y) = \dfrac{(e^{4y} + 1)(-\sin(10y) \cdot 10) - \cos(10y) \cdot 4e^{4y}}{(e^{4y} + 1)^2}$

(d) $s(z) = 2^{z^2 \sec(z)} \Rightarrow s'(z) = \ln(2)2^{z^2 \sec(z)} \cdot \left(z^2 \sec(z)\tan(z) + 2z\sec(z)\right)$

(e) $c(x) = \sin\left(e^{x^2}\right) \Rightarrow c'(x) = \cos\left(e^{x^2}\right) \cdot e^{x^2} \cdot 2x$

Day 2

A WARM-UP

Let's differentiate

$$f(x) = \cos(x^2 e^x).$$

ACTIVITY 2.5.4AB

Use known derivative rules, including the chain rule, as needed to answer each of the following questions.

(a) Find an equation for the tangent line to the curve $y = \sqrt{e^x + 3}$ at the point where $x = 0$. $y - 2 = \frac{1}{4}(x - 0)$

(b) If $s(t) = \dfrac{1}{(t^2 + 1)^3}$ represents the position function of a particle moving horizontally along an axis at time t (where s is measured in inches and t in seconds), find the particle's instantaneous velocity at $t = 1$. Is the particle moving to the left or right at that instant? $s'(1) = -\frac{6}{16} = -\frac{3}{8}$

ACTIVITY 2.5.4CD

(c) At sea level, air pressure is 30 inches of mercury. At an altitude of h feet above sea level, the air pressure, P, in inches of mercury, is given by the function

$$P = 30e^{-0.0000323h}.$$

Compute dP/dh and explain what this derivative function tells you about air pressure, including a discussion of the units on dP/dh. In addition, determine how fast the air pressure is changing for a pilot of a small plane passing through an altitude of 1000 feet.
$P'(h) = 30e^{-0.0000323h}(-0.0000323)$, so $P'(1) \approx -0.000938$ in Hg/ft

(d) Suppose that $f(x)$ and $g(x)$ are differentiable functions and that the following information about them is known:

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-1	2	-5	-3	4
2	-3	4	-1	2

If $C(x)$ is a function given by the formula $f(g(x))$, determine $C'(2)$. In addition, if $D(x)$ is the function $f(f(x))$, find $D'(-1)$. $C'(2) = -10$ and $D'(-1) = -20$

DESMOS

On your Desmos Student dashboard, find the activity called Derivative Practice and work on that.