§2.7: IMPLICIT DIFFERENTIATION

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Lecture 15

How can we calculate rates of change for functions given implicitly?

WHAT DOES THIS MEAN? (PREVIEW ACTIVITY DISCUSSION)

EXAMPLE (DESMOS)

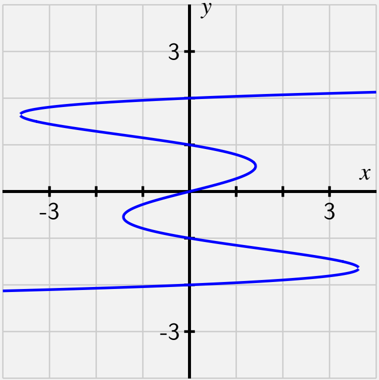
Let's find the line tangent to the curve

$$x^3 + xy + y^2 = 31$$

at the point $(1, 5)$.

ACTIVITY 2.7.2 (DESMOS)

Consider the curve defined by the equation $x = y^5 - 5y^3 + 4y$, whose graph is pictured in the figure.



(a) Explain why it is not possible to express y as an explicit function of x.

(b) Use implicit differentiation to find a formula for dy/dx. $\frac{dy}{dx} = \frac{1}{5y^4 - 15y^2 + 4}$

(c) Use your result from part (b) to find an equation of the line tangent to the graph of $x = y^5 - 5y^3 + 4y$ at the point $(0, 1)$.
$y - 1 = -\frac{1}{6}(x - 0)$

(d) Use your result from part (b) to determine all of the points at which the graph of $x = y^5 - 5y^3 + 4y$ has a vertical tangent line.

HORIZONTAL AND VERTICAL TANGENTS

If

$$\frac{dy}{dx} = \frac{p(x,y)}{q(x,y)}$$

the tangent line will be horizontal when:

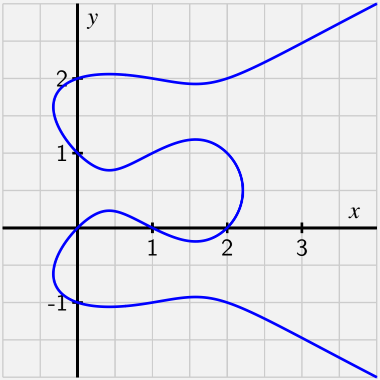
$$p(x,y) = 0 \text{ and } q(x,y) \neq 0$$

and vertical when:

$$q(x,y) = 0 \text{ and } p(x,y) \neq 0$$

ACTIVITY 2.7.3 (DESMOS)

Consider the curve defined by the equation $y(y^2-1)(y-2) = x(x-1)(x-2)$, whose graph is pictured in the figure.



Through implicit differentiation, it can be shown that

$$\frac{dy}{dx} = \frac{(x-1)(x-2) + x(x-2) + x(x-1)}{(y^2-1)(y-2) + 2y^2(y-2) + y(y^2-1)}.$$

(a) Determine all points (x, y) at which the tangent line to the curve is horizontal. (Use technology appropriately to find the needed zeros of the relevant polynomial function.)

(b) Determine all points (x, y) at which the tangent line is vertical. (Use technology appropriately to find the needed zeros of the relevant polynomial function.)

(c) Find the equation of the tangent line to the curve at one of the points where $x = 1$.

ACTIVITY 2.7.4

For each of the following curves, use implicit differentiation to find $\frac{dy}{dx}$ and determine the equation of the tangent line at the given point.

(a) $x^3 - y^3 = 6xy$ at $(-3, 3)$

We see $3x^2 - 3y^2\frac{dy}{dx} = 6x\frac{dy}{dx} + 6y$, so $\frac{dy}{dx} = \dfrac{3x^2 - 6y}{6x + 3y^2}$, and thus $\left.\frac{dy}{dx}\right|_{(-3,3)} = 1$. The tangent line is $y - 3 = 1(x + 3)$.

(b) $\sin(y) + y = x^3 + x$ at $(0, 0)$

We obtain $\cos(y)\frac{dy}{dx} + \frac{dy}{dx} = 3x^2 + 1$, so $\frac{dy}{dx} = \frac{3x^2+1}{\cos(y)+1}$, and thus $\left.\frac{dy}{dx}\right|_{(0,0)} = \frac{1}{2}$. The tangent line is $y = \frac{1}{2}x$.

(c) $3xe^{-xy} = y^2$ at $(0.619061, 1)$

We obtain $3e^{-xy} + 3xe^{-xy}\left(-x\frac{dy}{dx} - y\right) = 2y\frac{dy}{dx}$, so $\frac{dy}{dx} = \frac{3e^{-xy} - 3xye^{-xy}}{2y + 3x^2e^{-xy}}$, and thus $\left.\frac{dy}{dx}\right|_{(0.619061,1)} = 2.3878315652$. The tangent line is $y - 1 = 2.3878315652(x - 0.619061)$.