§3.5: GLOBAL OPTIMIZATION

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Lecture 20

How and when can we find the absolute/global extreme values of a function?

PREVIEW ACTIVITY DISCUSSION (DESMOS)

ACTIVITY 3.5.2 (DESMOS)

Let $g(x) = \frac{1}{3}x^3 - 2x + 2$.

(a) Find the critical numbers of g that lie in the interval $-2 \leq x \leq 3$.
$g'(x) = x^2 - 2$, so the critical numbers are $x = \pm\sqrt{2}$.

(b) Use a graphing utility to construct the graph of g on the interval $-2 \leq x \leq 3$.

(c) From the graph, determine the x-values at which the global max/min occur.

(d) How do our answers change if we consider the interval $-2 \leq x \leq 2$?

(e) What if we consider the interval $-2 \leq x \leq 1$?

THE EXTREME VALUE THEOREM

Theorem

A continuous function f on a domain $[a, b]$ attains an absolute minimum and maximum value on the interval.

Notes:

- Absolute min/max can occur at a critical number
- Absolute min/max can occur at the endpoint of the interval (if included)

So, find the critical numbers and evaluate the function at the critical numbers and endpoints. Pick the biggest and smallest values.

ACTIVITY 3.5.3

Find the *exact* absolute maximum and minimum of each function on the stated interval.

(a) $h(x) = xe^{-x}$, $[0, 3]$ The absolute maximum of h is e^{-1} and the absolute minimum is 0.

(b) $p(t) = \sin(t) + \cos(t)$, $[-\frac{\pi}{2}, \frac{\pi}{2}]$ The absolute maximum of p is $\sqrt{2}$ and the absolute minimum is -1.

(c) $q(x) = \frac{x^2}{x-2}$, $[3, 7]$ On $[3, 7]$ the absolute maximum of q is 9.8 and the absolute minimum is 8.

(d) $f(x) = 4 - e^{-(x-2)^2}$, $(-\infty, \infty)$ Absolute minimum at $x = 2$.

(e) $h(x) = xe^{-ax}$, $[0, \frac{2}{a}]$ $(a > 0)$

(f) $f(x) = b - e^{-(x-a)^2}$, $(-\infty, \infty)$, $a, b > 0$

ACTIVITY 3.5.4

A piece of cardboard that is 10×15 (each measured in inches) is being made into a box without a top. To do so, squares are cut from each corner of the box and the remaining sides are folded up. If the box needs to be at least 1 inch deep and no more than 3 inches deep, what is the maximum possible volume of the box? what is the minimum volume? Justify your answers using calculus.

(a) Draw a labeled diagram that shows the given information. What variable should we introduce to represent the choice we make in creating the box? Label the diagram appropriately with the variable, and write a sentence to state what the variable represents.

(b) Determine a formula for the function V (that depends on the variable in (a)) that tells us the volume of the box.
$V(x) = x(10 - 2x)(15 - 2x) = 4x^3 - 50x^2 + 150x$.

(c) What is the domain of the function V? That is, what values of x make sense for input? Are there additional restrictions provided in the problem? $1 \leq x \leq 3$

(d) Determine all critical numbers of the function V.
$x = \frac{25 \pm 5\sqrt{7}}{6} \approx 6.371459426, 1.961873908$.

(e) Evaluate V at each of the endpoints of the domain and at any critical numbers that lie in the domain.

- $V(1.961873908) = 132.0382370$
- $V(1) = 104$
- $V(3) = 108$

(f) What is the maximum possible volume of the box? the minimum? Max volume is 132.0382370 at $x = 1.961873908$; min volume is 104 at $x = 1$.