MESOPOTAMIAN MATHEMATICS

Dr. Mike Janssen

Lecture 3

OVERVIEW

1. Historical Context
2. Methods of Computation
3. Geometry
4. Square Roots and the Pythagorean Theorem
5. Solving Equations

Historical Context

Turkey
Taurus Mountains
Diyabakir
Gaziantep
Waśśukani
Mosul
Ninive
Mediterranean Sea
Aleppo
Khabur
Hatra
Assur
Kirkuk
Nuzi
Iran
Caspian Sea
Hama
MESOPOTAMIA
Tigris
Zagros Mountains
Mari
Palmyra
Lebanon
Syria
Beirut
Syrian Desert
Euphrates
Baghdad
Damaskus
Sippar
Babylon
Israel
Iraq
Kerbela
Kisch
Nadschaf
Nippur
Isin
Lagaš
Ahvaz
Jerusalem
Amman
Uruk
Charax-Spasinu
Ur
Basra
Jordan
approximate extent of the Persian Gulf until 5500 B.C.
Kuwait
Persian Gulf
300 km

UNITS

- Different units used
- Typically one “large” unit represented 60 small units
- Eventually adapted for system of numeration

Numeration and Computation

Grouping system based on 10:

- 1: 
- 10: 

So what does  represent?

For numbers greater than 59, Babylonians used a place value system based on powers of 60. What does

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represent?

We'll use 3,2,58 from now on.

ADDITION AND MULTIPLICATION

- Many existing tablets are multiplication tables
- No addition tables, so reasonable to assume that addition was understood/a reasonable algorithm existed. Perhaps:

$$23, 37 + 41, 32 = 1, 05, 09$$

- Multiplication tables: listed $n \times 1, \ldots, n \times 20$, then $n \times 30$, $n \times 40, 50$, so $7 \times 34 = 7 \times 30 + 7 \times 4$.

PRACTICE

In the Babylonian system:

- multiply 25 by $1,04$
- multiply 18 by $1,21$

SEXAGESIMAL FRACTIONS

- All multiplication tables found are for *regular* sexagesimal fractions
- After the sexagesimal "point": $1/60$, then $1/60^2$, etc.
- Reciprocal of 40 is $0;1,30$, or $\frac{1}{60}+\frac{30}{60^2}$
- Used for division by 40

Problem: Find the sexagesimal reciprocal of 18. Then divide 50 by 18.

REGULAR NUMBERS

Explore: What is the condition on n that ensures that it is a regular sexagesimal? That is, that its reciprocal is a finite sexagesimal fraction? base b?

Solution: The number n is regular if and only if the prime factors of n are some subset of 2, 3, and 5.

Geometry

DEVELOPMENT

- Grew out of need to survey land rather than accountancy
- Found procedures for finding square roots, Pythagorean triples, and “algebra”
- Like Egyptians, no symbols for operations or unknowns; instead a verbal mathematics

COEFFICIENTS OF A SHAPE

Consider an equilateral triangle of side length s.

- the altitude of the triangle is $h = \frac{\sqrt{3}}{2}s$
- the area of the triangle is $A = \frac{\sqrt{3}}{4}s^2$

The Babylonians approximate $\sqrt{3} \approx \frac{7}{4}$:

- For the altitude, they use the number $0;52,30$
- For the area, they use $0;26,15$

These were the coefficients of the equilateral triangle.

OTHER COEFFICIENTS

Given a circle of circumference C (the defining component):

- $d = C/3$
- $A = C^2/12$

Square Roots and the Pythagorean Theorem

SQUARE ROOTS

- Given in tables
- Problems chosen where possible so that square roots are rational
- Some irrational square roots are needed, particularly $\sqrt{2}$

TWO VALUES FOR $\sqrt{2}$

- Generally given as $1;25$
- Tablet YBC 7289: $1;24,51,10$

Problem: Convert each to decimals and determine the accuracy of the approximations.

Where did these approximations come from?

RADICAL ALGORITHM

Start with the identity $(x+y)^2 = x^2 + 2xy + y^2$.

To find $\sqrt{N}$:

- Choose a *regular* value of a close to, but less than N, so that $N - a^2$ is small. Set $b = N - a^2$.
- Then find c such that $2ac + c^2$ is as close as possible to b.Thus

$$\sqrt{N} \approx \sqrt{a^2 + b} \approx \sqrt{a^2 + 2ac + c^2} = a + c.$$

- If a^2 is sufficiently close to N, then $c^2 \ll 2ac$ so choose $c = (1/2)b(1/a)$.Therefore,

$$\sqrt{N} \approx a + (1/2)b(1/a).$$

THE ACTUAL APPROXIMATION

- Begin with $a = 1;20$
- Then $b = 0;13,20$ and $1/a = 0;45$, so

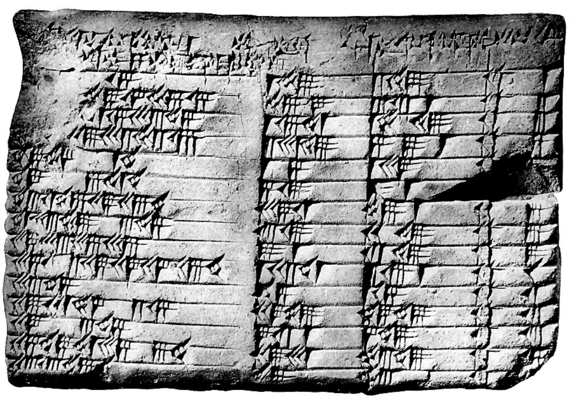
$$\sqrt{2} \approx 1;20 + (0;30)(0;13,20)(0;45) = 1;25.$$

PYTHAGOREAN TRIPLES

- Tablet Plimpton 322 (1800 BCE)
- Collection of Pythagorean triples, including

$$(12,709)^2 + (13,500)^2 = (18,541)^2$$

- Purpose of the values unclear
- Evidence of geometric understanding of the theorem elsewhere



Solving equations

OVERVIEW

- Lots of examples on tablets
- Equations like $ax = b$ solved by multiplying by $1/a$
- More complex equations/systems solved via false position

Example (VAT 8389)
One of two fields yields 2/3 *sila* per *sar*, the second yields 1/2 *sila* per *sar*. The yield of the first field was 500 *sila* more than that of the second, and the areas of the two fields together were 1800 *sar*. How large is each field?

From Tablet BM 13901:

Example
"I summed the area and two-thirds of my square-side and it was $0;35$. You put down 1, the projection. Two-thirds of 1, the projection, is $0;40$. You combined its half, $0;20$ and $0;20$. You add $0;06,40$ to $0;35$ and $0;41,40$ squares $0;50$. You take away $0;20$ that you combined from the middle of $0;50$ and the square-side is $0;30$."

CONCLUSION

- What we have of Egyptian and Babylonian mathematics are generally teaching documents–basically a set of example-types.
- Learning math was essentially learning how to select and modify an appropriate algorithm, and master the techniques required to carry out the algorithm. Reasoning was evidently transmitted orally.
- "Real-world" quadratic problems are just as contrived as most found in current school algebra texts.
- Thus, it's reasonable to conclude that solving the quadratic wasn't the important thing–training young minds for problem-solving was.