# MATH 390: ANCIENT AND MEDIEVAL CHINA

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Lecture 9

# TODAY'S PLAN

- Historical Context
- Number and calculation
- Geometry
- Algebra
- Remainder theorem

# Context

# CHINESE CIVILIZATION AND MATHEMATICAL TRAINING

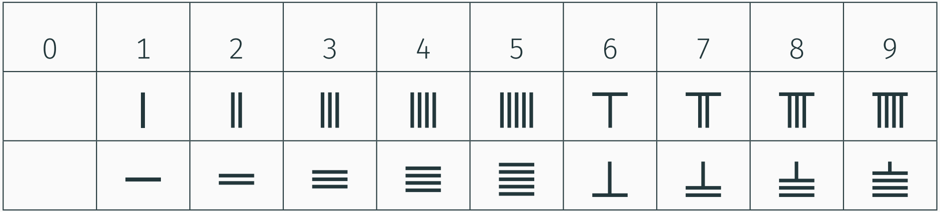
- Legends date Chinese civilization back 5000 years; evidence for 3600 years (Shang dynasty)
- Sixth century BCE: feudal period, Confucius, flowering of scholarship
- Standardization of weights, measures, money, writing
- Han dynasty (202 BCE-220 CE): imperial civil service examinations that lasted until the 20th century
- Li Chunfeng: collected and annotated the *Ten Mathematical Classics*: *Arithmetical Classic of the Gnomon*, *The Nine Chapters*, *Sea Island Mathematical Manual*, etc.
- Colletions of problems with methods of solution; examinations required recitations of the texts
- No incentive for mathematical creativity

Shang Dynasty, graphic by Lamassu Design

# Calculations

# NUMBER SYMBOLS

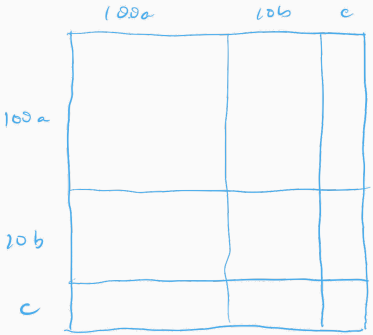
- Base 10
- Shang dynasty: elaborate symbols written multiplicatively
- Counting rods, manipulated on a counting board:




See p. 198 of the text

# FRACTIONS

- E.g.: 2/3 represented as “3 *fen zhi* 2”, meaning “2 parts from a whole broken into 3 equal parts”
- Rule for reducing fractions to lowest terms (p. 198): essentially the Euclidean algorithm
- Rule for adding fractions: try doubling, tripling, etc; if all else fails, multiply the denominators




Let's calculate $\sqrt{53,824}$ by finding digits $a, b, c$ such that $(100a + 10b + c)^2 = 53824$.

1. First, find the largest $a$ so that $(100a)^2 < 53824$. We see $a = 2$.
2. Difference is 13824, so $b$ must satisfy $13824 > 2(100a)(10b) = 4000b$, which suggests $b = 3$. Check by verifying that $2(100a)(10b) + (10b)^2 < 13824$, which it is.
3. Repeat to find $c = 2$, so $\sqrt{53824} = 232$.

# Geometry

# AREA OF A CIRCLE

- Lots of formulas for areas and volumes of geometrical figures
- Problem 32 from the first chapter of *The Nine Chapters*:
  *There is a round field whose circumference is 181 yards and whose diameter is 60 1/3 yards. What is the area of the field? Answer: 2730 1/12 square yards.*

Note:

- $C/d = 3$
- Four formulas given, e.g: The diameter is multiplied by itself. Multiply the result by 3 and then divide by 4.
- Also: The circumference is multiplied by itself. Then divide the result by 12. Babylonians!

# PYTHAGOREAN THEOREM AND APPLICATIONS

- Ancient document seem to assume knowledge of the Pythagorean theorem.
- Commentaries on, e.g., *The Nine Chapters* give an argument and reference diagrams which have been lost.
- Not proofs according to modern standards (or even Euclid), but the Chinese found diagrams convincing.
- Liu Hui's commentary (3rd century) on Ch. 9 became the *Sea Island Mathematical Manual*.

**Problem**

*For the purpose of looking at a sea island, erect two poles of the same height, 5 feet, the distance between the front and rear pole being 1000 feet. Assume that the rear pole is aligned with the front pole. Move away 123 feet from the front pole and observe the peak of the island from ground level. Move backward 127 feet from the rear pole and observe the peak of the island from ground level again; the tip of the back pole also coincides with the peak. What is the height of the island and how far is it from the front pole?*

# Algebra

Two basic methods:

- Surplus and deficiency: make two guesses and scale
- Method 2: From Ch. 8 of the *Nine Chapters*, Problem 1:

**Problem**

*There are three classes of grain, of which three bundles of the first class, two of the second, and one of the third make 39 measures. Two of the first, three of the second, and one of the third make 34 measures. And one of the first, two of the second, and three of the third make 26 measures. How many measures of grain are contained in one bundle of each class?*

# SOLUTION

$$\begin{matrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 1 & 1 \\ 26 & 34 & 39 \end{matrix} \rightarrow \begin{matrix} 1 & 0 & 3 \\ 2 & 5 & 2 \\ 3 & 1 & 1 \\ 26 & 24 & 39 \end{matrix} \rightarrow \begin{matrix} 0 & 0 & 3 \\ 4 & 5 & 2 \\ 8 & 1 & 1 \\ 39 & 24 & 39 \end{matrix} \rightarrow \begin{matrix} 0 & 0 & 3 \\ 0 & 5 & 2 \\ 36 & 1 & 1 \\ 99 & 24 & 39 \end{matrix}$$

# Remainder Theorem

**Problem (Sunzi suanjing)**
*We have things of which we do not know the number; if we count them by threes, the remainder is 2; if we count them by fives, the remainder is 3; if we count them by sevens, the remainder is 2. How many things are there?*

Find $N$ satisfying

$$N = 3x + 2 \qquad N = 5y + 3 \qquad N = 7z + 2 \qquad \text{for some } x, y, z \in \mathbb{Z}.$$

Or:

$$N \equiv 2 \mod 3 \qquad N \equiv 3 \mod 5 \qquad N \equiv 2 \mod 7$$

# SUN ZI'S SOLUTION

"If you count by threes and have the remainder 2, put 140. If you count by fives and have the remainder 3, put 63. If you count by sevens and have the remainder 2, put 30. Add these numbers and you get 233. From this subtract 210 and and you get 23."

# TRANSMISSION TO AND FROM CHINA

- Not much known about transmission of mathematical ideas to/from China before the 1500s
- Many techniques and ideas across China, India, Europe, and the Islamic world
- Empty place on the counting board in China, decimal place value, European discovery of the Chinese remainder problem
- Multiple discovery? Not quite.
- Jesuit priest Mateo Ricci came to China in the late 1500s; he and a Chinese student, Xu Guangqi translated the first six books of the *Elements* in 1607.
- Western math thus entered China and indigenous math began to disappear