MATH 390: INDIAN ARITHMETIC AND ALGEBRA

Dr. Mike Janssen

Lecture 10

TODAY'S PLAN

- Historical Context
- Number and calculation
- Geometry
- Algebra
- Astronomy, Trigonometry, and Approximations

Context


CHINA
Yarlung Zangbo (Tsangpo)
Lhasa
NEPAL
Kathmandu
BHUTAN
Subansiri
Lohit
Brahmaputra
Guwahati
New Delhi
Agra
Jaipur
Yamuna
Ganges
Lucknow
Ghaghara
Gandak
Kosi
Teesta
Kanpur
Gomti
Ganges
Patna
Chambal
Banas
Betwa
Allahabad
Varanasi
Tamsa
Son
Farakka
BANGLADESH
Surma
Barak
Jamuna
Dhaka
Damodar
Hooghly
Padma
Meghna
Indore
Bhopal
Kolkata
Mouths of the Ganges
INDIA
MYANMAR
Bay of Bengal

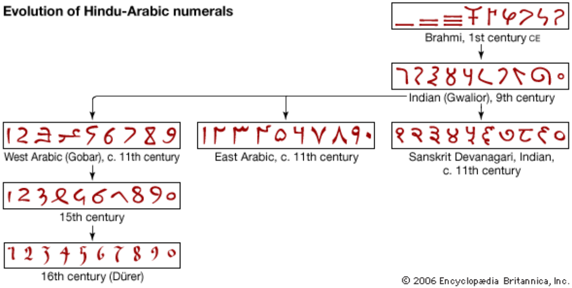
Calculations

DIGITS

- Three important components of the decimal place value system:
  - digits 1-9
  - notion of place value
  - use of 0
- Actual history of these components are lost
- The digits themselves date at least to King Ashoka (mid-third century BCE) in India; likely transmitted to Europe via Islamic incursions into northern India

PLACE VALUE

- Early Indian evidence suggests symbols for 1-9 as well as 10-90
- Around 600, just used symbols for 1-9 in our familiar place value arrangement
- Earliest reference from Syrian priest Severus Sebokht (662): Hindus have a valuable method of calculation “done by means of nine signs.”
- Evidence of a dot to represent 0 also dates to 7th century CE, but perhaps Severus did not consider it a sign.
- Why did they drop to only 9 signs?
- Perhaps influence from Chinese counting boards
- No evidence of decimal fractions in India; this came from the Islamic empire


Evolution of Hindu-Arabic numerals
Brahmi, 1st century CE
Indian (Gwalior), 9th century
West Arabic (Gobar), c. 11th century
East Arabic, c. 11th century
Sanskrit Devanagari, Indian, c. 11th century
15th century
16th century (Dürer)
© 2006 Encyclopædia Britannica, Inc.

A NOTE ON CALCULATIONS

- Had methods of calculating cube/square roots
- In *The Correct Astronomical System of Brahma*, Brahmagupta gave the standard arithmetical rules for calculating with fractions
- Gave rules for calculating with 0 and negative numbers as well: *The product of a negative and a positive is negative, of two negatives positive, and of positives positive; the product of zero and a negative, of zero and a positive, or of two zeros is zero. A positive divided by a positive or a negative divided by a negative is positive; a zero divided by a zero is zero; a positive divided by a negative is negative; a negative divided by a positive is also negative. A negative or a positive divided by zero has that zero as its divisor, or zero divided by a negative or a positive has that negative or positive as its divisor.*

Geometry

SULBASUTRA

- Dates to 600 BCE
- Geometric results for constructing fire-altars
- Not designed to teach mathematics; no demonstrations
- No axiomatic method
- Some commentaries contain proofs of a sort
- Contains the Pythagorean theorem, in turn used to justify several constructions
- Other results on circles; $\pi = 4(13/15)^2 = 3.00444444$.

Solving Equations

QUADRATICS

- Brahmagupta, for the equation $ax^2 + bx = c$:

  > *Diminish by the middle number the square root of the rūpas multiplied by four times the square and increased by the square of the middle number; divide the remainder by twice the square. The result is the middle number.*

  This yields:

$$x = \frac{\sqrt{4ac + b^2} - b}{2a}.$$

- If an equation had a negative solution, it was not mentioned.
- Bhaskara II (approx 1150 CE) did deal with multiple roots when they were positive via completing the square

Problem (Mahāvīra)
Doves are sold at the rate of 5 for 3 coins, cranes at the rate of 7 for 5, swans at the rate of 9 for 7, and peacocks at the rate of 3 for 9. A certain man was told to bring at these rates 100 birds for 100 coins for the amusement of the king's son and was sent to do so. What amount does he give for each?

$$3d + 5c + 7s + 9p = 100$$
$$5d + 7c + 9s + 3p = 100$$

- Could solve systems of congruences like

$$N \equiv a \mod r$$
$$N \equiv b \mod s.$$

- Likely did not learn this from elsewhere (unlike quadratics) as the method is wholly original.

Example (Brahmagupta)
Solve

$$N \equiv 10 \mod 137$$
$$N \equiv 0 \mod 60.$$

Equivalent to the equation $137x + 10 = 60y$.

Trigonometry

ASTRONOMY

- Evidence of Greek astronomical knowledge transmitted to India, likely via Roman trade routes
- Needs of each dictated the development of their astronomies
- Early fifth century: *Paitāmahasiddhānta*, contains a table of "half-chords"
- We define

$$\theta = R \sin \theta.$$

- Cosines tabulated in the sixth century
- Purpose: use, e.g., shadows, to determine the time

NEED FOR APPROXIMATION TECHNIQUES

- Goal was in part to approximate Sine with rational functions
- For a long time, the accuracy of algebraic/interpolation schemes were sufficient for their uses
- As sea navigation increased, quicker and more accurate calculations for latitude and longitude became necessary
- (Bhāskara I) Near the equator, one had to determine latitude ϕ by observation of the solar altitude at noon, μ:

$$R\delta = \phi\mu,$$

where δ is the sun's declination (known from tables)

APPROXIMATING SINE AND COSINE

(Jyesthadeva) Around the late fourteenth century, the following approximations appear:

$$\cos s \approx 1 - \frac{s^2}{2} + \frac{s^4}{24}$$
$$\sin s \approx s - \frac{s^3}{6} + \frac{s^5}{120}.$$

Since Jyesthadeva considered each new term a correction to the previous value, he understood that the more terms taken, the more closely the polynomials approach the true value for sine and cosine.

Thus, the Indians had discovered the sine and cosine power series.

TRANSMISSION TO/FROM INDIA

- Indians learned astronomy and trigonometry from Greek sources
- Islamic scholars learned trigonometry when Indian works brought to Baghdad in the eighth century
- Our decimal system originated in India and traveled to western Europe over the course of several hundreds of years
- How did the quadratic formula make its way to/from India (if indeed it did)?
- No available documentation of the power series for sine/cosine making it to Europe before they figured it out in the mid-1600s
- Possibility: Jesuits collected and translated mathematical texts in South India and brought them back
- No reason to think Newton knew of power series, but other European mathematicians may have