MATH 390: ISLAMIC MATHEMATICS

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INTRODUCTION AND CONTEXT

- Islam founded in early 600s
- Muhammad captures Mecca in 630; Islam enters Spain in 711
- Caliph al-Mansūr founded Baghdad in 766
- Increase in standard of living and tolerance of others allowed for intellectual flourishing
- Caliph al-Ma'mūn (813-833) established the House of Wisdom, a research institute, in Baghdad

ISLAMIC MATHEMATICS

- Arabic language spoken, but influence of Islamic religion extremely important
- Islamic mathematicians felt "secular knowledge" was a way to deeper "holy knowledge" and so learning and research was encouraged, at least until the 11th century
- Then attitudes changed; "foreign sciences" like mathematics seen as subversive by many religious leaders
- By the end of the 9th century, most principal Greek mathematical works translated into Arabic and gathered at the House of Wisdom
- Scholars also learned Hindu math and absorbed the local Babylonian math still extant in the Tigris-Euphrates valley

Decimal Arithmetic

CALCULATIONS

- Al-Khwārizmī (780-850) wrote a text on Hindu calculation methods
- Introduced symbols for 1-9, a circle for 0, and place value
- Al-Uqlīdīsī wrote a text in which he gave a pitch for the Indian approach as being "easy, quick".
- Advantage of Al-Uqlīdīsī's text is that he showed the steps for multiplying large numbers
- Also treated decimal fractions–the earliest recorded instance outside of China
- Not certain that al-Uqlīdīsī understood what he was doing

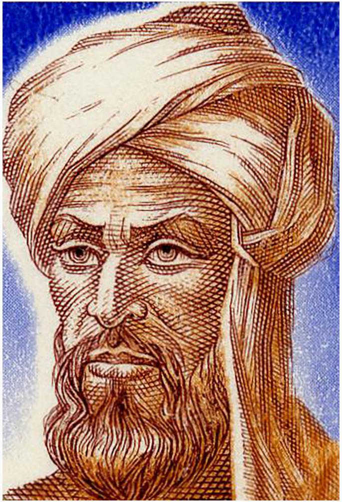
Algebra

ISLAMIC ALGEBRA

- By far the most important/influential contribution
- Began abstracting algebraic questions from geometric
- Combined Babylonian ideas with Greek, especially in proof
- Believed that a solution was not valid until it was demonstrated so, via geometric proof
- Everything was verbal; no symbolism whatsoever

MUHAMMAD IBN AL-KHWĀRIZMĪ

- From modern-day Uzbekistan
- Influenced by the Greeks at the House of Wisdom
- *The Condensed Book on the Calculation of al-Jabr and al-Muqabala*
- *al-jabr*: restoring/transposing
- *al-muqabala*: comparing
- Introduction mentions ‘usefulness’ but that’s a stretch



CLASSIFYING QUANTITIES AND EQUATIONS

Three types of quantities:

- The square (of the unknown)
- The root of the square (the unknown itself)
- The absolute number (constant in the equation)

Six types of equations:

1. Squares equal to roots: $ax^2 = bx$
2. Squares equal to numbers: $ax^2 = c$
3. Roots equal to numbers: $bx = c$
4. Squares and roots equal to numbers: $ax^2 + bx = c$
5. Squares and numbers equal to roots: $ax^2 + c = bx$
6. Roots and numbers equal to squares: $bx + c = ax^2$

EXAMPLE

Problem

What must be the square which, when increased by ten of its roots, amounts to thirty-nine?

Solution: The solution is this: you halve the number of roots, which in the present instance yields five. This you multiply by itself; the product is twenty-five. Add this to thirty-nine; the sum is sixty-four. Now take the root of this which is eight, and subtract from it half the number of the roots, which is five; the remainder is three. This is the root of the square which you sought for.

Ibn Qurra and Abū Kāmil

- Al-Khwārizmī's work with quadratics rested on Babylonian ideas
- In the late 9th century, ibn Qurra and Abū Kāmil put Al-Khwārizmī's work on the foundations of Euclid
- Abū Kāmil also used irrationals freely; he solves an equation with solution

$$x = 10 + \sqrt{2} - \sqrt{42 + \sqrt{800}}$$

- Also solved systems of equations (not just linear), divides by unknown quantities, and use algebraic algorithms with any type of positive 'number'
- All without symbols

The Algebra of Polynomials

- Worked in Baghdad around the year 1000
- First to realize that powers of a variable could be extended indefinitely
- Established general procedures for arithmetic with powers of a variable (and their reciprocals)
- Proved $1^3 + 2^3 + 3^3 + \cdots + 10^3 = (1 + 2 + 3 + \cdots + 10)^2$ by induction

AL-SAMAW'AL

- Born to Jewish parents in Baghdad (1125-1174)
- Converted to Islam at age 40; autobiography stating reasons for conversion became the basis for polemical writings against Jews
- Wrote his major mathematical work, *Al-Bāhir*, when he was 19
- Established rules for adding and subtracting polynomials by combining like terms; fundamental in extending properties of numbers to variables
- Explained the law of exponents: $x^m x^n = x^{m+n}$
- Willing to extend polynomial division into polynomials in $1/x$
- Proved the binomial theorem (by induction), COMPLETELY VERBALLY

OMAR KHAYYAM AND THE CUBIC

- Born in Nishapur, Iran (1048-1131)
- Certain Greek problems led to cubic equations (e.g., doubling the cube)
- Was able to solve some cubics by intersecting conics
- Motivated by algebraic, not geometric problems
- Could solve certain cubics, but wanted a general approach like Al-Khwārizmī for quadratics

KHAYYAM'S CLASSIFICATION OF CUBICS

Binomial:

1. $x^3 = d$

Trinomial:

2. $x^3 + cx = d$
3. $x^3 + d = cx$
4. $x^3 = cx + d$
5. $x^3 + bx^2 = d$
6. $x^3 + d = bx^2$
7. $x^3 = bx^2 + d$

Tetranomial:

8. $x^3 + bx^2 + cx = d$
9. $x^3 + bx^2 + d = cx$
10. $x^3 + cx + d = bx^2$
11. $x^3 = bx^2 + cx + d$
12. $x^3 + bx^2 = cx + d$
13. $x^3 + cx = bx^2 + d$
14. $x^3 + d = bx^2 + cx$

Evidence of a shift in thinking; interest in solving cubics sparked by geometry, but has gone beyond it

FINDING AND COUNTING SOLUTIONS

- Khayyam analyzed each of the fourteen cases for 0, 1, or 2 positive solutions, with only one error; did not notice that $x^3 + cx = bx^2 + d$ can have three positive solutions
- Sharaf al-Dīn al-Tūsī (d. 1213) later determined conditions on the coefficients that would count the number of (positive) solutions
- Effectively identified the cubic discriminant