# MATH 390: MATHEMATICS IN MEDIEVAL EUROPE

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Lecture 12

# From East to West

# WHERE WE'VE BEEN

- 2000BC–600BC: Mathematics for calculation, government management, etc.
- 600BC-400AD: Flourishing of Greek mathematics, especially geometry and the axiomatic method

Meanwhile:

- Chinese mathematics: parallel, isolated track until the 1500s or so
- Indian mathematics: Developed numeration, some geometry, algebra and number theory; crossroads between west and east
- Islamic mathematics: adapted Hindu numeration; married Greek proof to algebraic questions

# CONTEXT: MEDIEVAL EUROPE

- Time: 5th-late 15th centuries
- First 500 years or so: level of cultural development was low
- Self-sufficient feudal estates throughout most of Europe
- Little trade after the Muslim conquest of the Mediterranean sea routes

# THE TRIVIUM AND QUADRIVIUM

- In this context: what is required for an educated person?
- Inherited from Greeks: the TRIVIUM and QUADRIVIUM
- The TRIVIUM: grammar, logic, and rhetoric
- The QUADRIVIUM: arithmetic, geometry, music, and astronomy
- Embraced by the Roman Church:

  *[W]e must not despise the science of numbers, which, in many passages of Holy Scripture, is found to be of eminent service to the careful interpreter. Neither has it been without reason numbered among God's praises: 'Thou hast ordered all things in number, and measure, and weight.'*

  –Augustine, CITY OF GOD
- Unfortunately, the only texts available for the study of the quadrivium were brief introductions

# A SIGNIFICANT EARLY PROBLEM

- The calendar
- Church debate: should Easter be determined using the Roman solar calendar or the Jewish lunar calendar?
- They could be reconciled, but only with some mathematical knowledge
- Charlemagne formally recommended that the mathematics necessary for Easter computations be a part of the curriculum in Church schools

| Variable | Expression | *year* = 1777 | 2021 |
|---|---|---|---|
| $a =$ | *year* mod 19 | 10 | 7 |
| $b =$ | *year* mod 4 | 1 | 1 |
| $c =$ | *year* mod 7 | 6 | 5 |
| $k =$ | *year* div 100 = $\lfloor \frac{year}{100} \rfloor$ | 17 | 20 |
| $p =$ | $(13 + 8k)$ div 25 = $\lfloor \frac{13+8k}{25} \rfloor$ | 5 | 6 |
| $q =$ | $k$ div 4 = $\lfloor \frac{k}{4} \rfloor$ | 4 | 5 |
| $M =$ | $(15 - p + k - q) \bmod 30$ | 23 | 24 |
| $N =$ | $(4 + k - q) \bmod 7$ | 3 | 5 |
| For the Julian Easter in the Julian calendar $M = 15$ and $N = 6$ ($k$, $p$ and $q$ are unnecessary) | | | |
| $d =$ | $(19a + M) \bmod 30$ | 3 | 7 |
| $e =$ | $(2b + 4c + 6d + N) \bmod 7$ | 5 | 6 |
| March Easter day = | $22 + d + e$ | 30 | 35 |
| April Easter day = | $d + e - 9$ | -1 | 4 |
| | $(11M + 11) \bmod 30$ | 24 | 5 |
| if $d = 28$, $e = 6$, and $(11M + 11) \bmod 30 < 19$, replace 25 April with 18 April | | | |
| if $d = 29$ and $e = 6$, replace 26 April with 19 April | | | |

https://en.wikipedia.org/wiki/Date_of_Easter

# INTO THE NEW MILLENNIUM

- Gerbert d'Aurillac (Pope Sylvester II): studied in Spain and learned mathematics of the Muslims
- Reintroduced the study of mathematics into the cathedral schools: arithmetic and geometry, first Western use of Hindu-Arabic numerals (on a counting board)
- Toledo, Spain, taken by Christians in 1085: Islamic texts had been translated to Spanish, and then eventually into Latin (see list on p. 327)
- Robert of Chester translated al-Kwārizmī's *Algebra* in 1145 and introduced it to Europe
- Most major Greek mathematical works translated to Latin by the end of the 12th century

# Geometry

# PRACTICAL GEOMETRY

- Abraham bar Hiyya (1116): help French and Spanish Jews with the measurements of their fields
- Begins with a summary of important aspects of Euclid

*circles!*

- Hugh of St. Victor (1096-1141): theologian and master of the abbey of St. Victor in Paris
- Wrote a practical geometry text, designed for surveyors, no mention of Euclid or trigonometry
- Euclid reached Paris by the late 12th century
- Leonardo of Pisa's *Practica Geometriae* (1220): section on rectangles includes results on quadratics; approximated $\pi \approx 22/7$ using Archimedes' inscription of a 96-sided polygon in a circle

# Combinatorics

# "THE BOOK OF CREATION"

- Indian and Islamic mathematicians were interested in combinatorial questions
- Earliest Jewish source: the *Sefer Yetsirah* (Book of Creation), author unknown
- Calculated the various ways in which the 22 letters of the Hebrew alphabet could be arranged
- Believed that God had created everything by naming them
- Thus, knowing the number of ways to create words was of interest:

  > *Two stones [letters] build two houses [words], three build six houses, four build twenty-four houses, five build one hundred and twenty houses, six build seven hundred and twenty houses, seven build five thousand and forty houses.*

# ABRAHAM IBN EZRA

- Spanish-Jewish philosopher, astrologer, and biblical commentator
- Discussed the number of possible conjunctions of the seven 'planets' (sun and moon included)
- Thus calculated $\binom{7}{k}$ for $k = 2, 3, \ldots, 7$.
- Noted symmetry: $\binom{7}{2} = \binom{7}{5}$
- Also introduced Hebrew-speaking community to decimal place values

# INDUCTION

- Levi ben Gerson: The Art of the Calculator (1321)
- Modern-sounding justification for learning theoretical mathematics
- Included a technique he called "rising step by step without end"
- In general, first proves the inductive step, then notes that the process begins at some small value of $k$

**Proposition (Propositions 9 and 10)**

- *If one multiplies a number which is the product of two numbers by a third number, the result is the same as when one multiples the product of any two of these three numbers by the third.*
- *If one multiplies a number which is the product of three numbers by a fourth number, the result is the same as when one multiplies the product of any three of these four numbers by the fourth.*

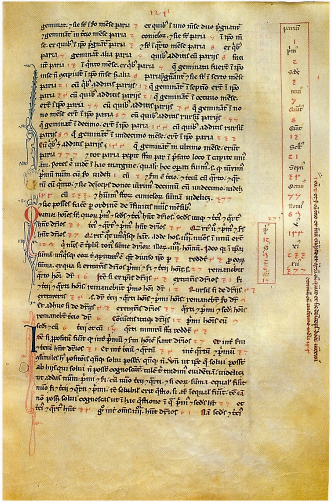
# Medieval Algebra

# LEONARDO OF PISA

- 1170-1250, Republic of Pisa
- Traveled as a young boy and was educated in Hindu-Arabic numerals
- Father directed a trading post in what is now Algeria
- Published the *Liber Abbaci* (*Book of Calculation*) in 1202

# LIBER ABBACI

- Sources largely in the Islamic world
- Contains rules for calculating with Hindu-Arabic numerals
- Practical problems around calculation of profits, currency conversions, measurement
- Includes mixture problems, motion problems, Chinese remainder problem, and problems solvable by quadratics
- Limited theory
- Often chooses special procedures over general methods



# EXAMPLE: BUYING BIRDS

PROBLEM: How can one buy 30 birds for 30 coins, if partridges cost 3 coins each, pigeons 2 coins each, and sparrows 2 for 1 coin?

# EXAMPLE: LION IN THE PIT

PROBLEM: A lion is in a pit which is 50 feet deep. The lion climbs up 1/7 of a foot each day and then falls back 1/9 of a foot each night. How long will it take him to climb out of the pit?

# EXAMPLE: RABBITS

*How many pairs of rabbits are created in one year? A certain man had one pair of rabbits together in an enclosed place, and one wishes to know how many are created from the pair in one year when it is the nature of them in a single month to bear another pair, and in the second month those born to bear also.*

# Jordanus de Nemore

- Believed to have taught in Paris around 1220
- Speculation that Jordanus was a pseudonym, possibly for a woman
- Writings on arithmetic, geometry, astronomy, mechanics, and algebra
- One of the first to make advances over the work of Fibonacci
- In contrast with Boethius's demonstrationless arithmetic, the ten books of the *Arithmetica* were inspired by Euclid, with definitions, axioms, postulates, propositions, and careful proofs
- Also like Euclid: no numerical examples
- Theoretical work of arithmetic for the quadrivium
- Not entirely rhetorical: use of letters to stand for arbitrary numbers

# SELECTED PROPOSITIONS

**Proposition (I-1)**

*If a given number is divided into two parts whose difference is given, then each of the parts is determined.*

**Proposition (IV-9)**

*If the square of a number added to a given number is equal to the number produced by multiplying the root and another given number, then two values are possible.*

That is, there are two solutions to $x^2 + c = bx$.