MATH 390: ALGEBRA IN THE RENAISSANCE

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Lecture 13

THE ITALIAN ABACISTS

- Middle Ages merchants: traveled themselves to distant places, bought and brought back goods to sell
- Increased safety of sea travel in the Renaissance meant that they didn't have to go themselves
- This resulted in a need for more complex mathematics, e.g., dealing with lines of credit, interest calculations, etc.
- Increased need for algebra, and techniques for solving more complex equations
- Slow shift from Roman to Hindu-Arabic numerals *check-writing!*

SYMBOLISM

- Early in the 15th century, some abbreviations began to be substituted for standard words.
  - *cosa: c*
  - *censo: ce*
  - *cubo: cu*
  - *radice: R*
- Then, e.g,. the fourth power, or *censo di censo* became *ce ce*

HIGHER-DEGREE EQUATIONS

- Italian abacist work traditionally began with a presentation of al-Khwārizmī's six types of linear and quadratic equations
- Extended in 1344 by Maestro Dardi of Pisa to 198 types of equations of degree ≤ 4, some of which involved radicals
- Generally reduced to some previously-known case

Example

A man lent 100 lire to another and after 3 years received back a total of 150 lire in principal and interest, where the interest was compounded annually. What was the interest rate?

The Cubic Formula

BACKGROUND AND CONTEXT

- Quadratic formula: solve an equation purely in terms of the coefficients and arithmetic operations
- Noted as late as 1494 that there was no known algebraic solution to the general cubic $ax^3 + bx^2 + cx + d = 0$.
- Known that one could reduce a general cubic to the so-called DEPRESSED cubic

$$x^3 + px + q = 0$$

- Thus, solving the depressed cubic would yield a general solution.

SCIPIONE DEL FERRO'S SOLUTION

- 1465-1526
- Professor at the University of Bologna
- Discovered an algebraic method of solving $x^3 + cx = d$
- Negative coefficients still not allowed, so this is but one of 13 cases
- Modern v. Renaissance academia
- Did disclose his solution to his student, Antonio Maria Fiore, and successor/son-in-law Annibale della Nave

FONTANA

- Word began to circulate that the cubic would soon be solved
- Niccolò Fontana (1499-1557, aka Tartaglia, the “stutterer”) boasted that he had solved cubics of the form $x^3 + bx^2 = d$
- 1535: Fiore challenged Tartaglia to a public contest, posing 30 questions about cubics of the form $x^3 + bx = c$
- Tartaglia worked and discovered the general solution to this case; Fiore was unable to solve many of Tartaglia’s problems and lost the competition

GEROLAMO CARDANO (1501-1576)

- Public lecturer in mathematics
- Wrote to Tartaglia, wanting to include his solution to the cubic in a new arithmetic text Cardano was writing
- Tartaglia eventually relented and came to Milan
- Cardano pledged an oath not to publish Tartaglia’s solution
- Tartaglia gave the solution in a poem

TARTAGLIA'S POEM

For $x^3 + cx = d$:

When the cube and its things near
Add to a new number, discrete,
Determine two new numbers different
By that one; this feat
Will be kept as a rule
Their product always equal, the same,
To the cube of a third
Of the number of things named.
Then, generally speaking,
The remaining amount
Of the cube roots of subtracted
Will be your desired count.

THE ARS MAGNA

- Cardano kept his promise not to publish Tartaglia's result in the new arithmetic book
- Cardano began to work on the problem himself, assisted by his student Lodovico Ferrari (1522-1565)
- Worked out all the cases in the coming years; Tartaglia still hadn't published
- Cardano heard a rumor that the original solution had been found by del Ferro in Bologna, so he went to see della Nave, who showed Cardano del Ferro's notes
- Cardano no longer felt an obligation to Tartaglia; instead he'd publish del Ferro's solution, discovered 20 years earlier
- 1545: the *Ars Magna* is published, and includes solutions to the cubic and quartic

Cardano's Formula

EXAMPLES

Find a solution to

- $x^3 - 19x + 30 = 0$
- $x^3 - 15x - 4 = 0$

What does it mean to take square roots of negative numbers?