MATH 390: DEVELOPMENTS IN ALGEBRA

BOMBELLI, VIÈTE, AND STEVIN

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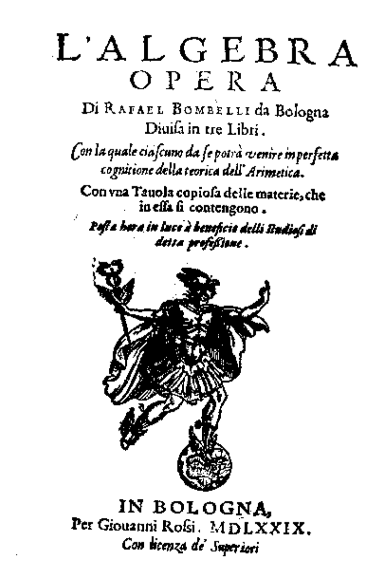
Lecture 14

Bombelli

RAFAEL BOMBELLI (1526-1572)

- From Bologna
- Wrote a systematic text to enable students to master Cardano's algebra
- Began the book with elementary material and worked up to solving the cubic and quartic
- First European to write down symbols for performing computations with negative numbers.
- Bombelli introduced different symbolism than Cardano had used



L'ALGEBRA
OPERA
Di Rafael Bombelli da Bologna
Diuisa in tre Libri.
Con la quale ciascuno da se potrà venire in perfetta cognitione della teorica dell'Arimetica.
Con vna Tauola copiosa delle materie, che in essa si contengono.
Posta hora in luce à beneficio delli Studiosi di detta professione.
IN BOLOGNA,
Per Giouanni Rossi. MDLXXIX.
Con licenza de' Superiori

BOMBELLI'S SYMBOLS

- *R.q.*: square root
- *R.c.*: cube root
- $\lfloor\ \rfloor$: grouping
- Major innovation: semicircle around a number to denote a power of the unknown

A NEW CUBE ROOT

- Part I of the *Algebra*: a new type of cube root "much different from the former" which comes from equations of the form $x^3 = cx + d$.
- This root "has its own algorithms for various operations and a new name."
- Called *più di meno* and *meno di meno*: "plus of minus" and "minus of minus"
- $2\ p\ di\ m\ 3 = 2 + 3i$

RULES AND CONSEQUENCES

- plus of minus times plus of minus gives minus
- plus of minus times minus of minus gives plus
- "the whole matter seems to rest on sophistry rather than on truth": didn't really believe they were meaningful numbers, but defined consistent rules via analogy to real numbers
- Uses them to discuss Cardano's formula in more cases
- Also used them to solve more quadratics, e.g., $x^2 + 20 = 8x$
- Last Italian algebraist of the Renaissance; his *Algebra* was widely read elsewhere in Europe

Viète

SIXTEENTH CENTURY ALGEBRA

- Continuation of Islamic algebra had reached its peak
- Many authors used symbolism for unknowns and its powers, but techniques were illustrated via example, as no symbols for coefficients were in use
- Renewed effort at clear, accurate translations of Greek mathematicians
- Attempts to understand how Greeks got their theorems

FRANÇOIS VIÈTE (1540-1603)

- Lawyer in Fontenay-le-Comte, western France
- Cryptanalyst for King Henri III in 1589
- Explored Greek ANALYSIS: assume "that which is sought as if it were admitted and working through the consequences of that assumption to what is admittedly true"

NEW SYMBOLISM

- INTRODUCTION TO THE ANALYTIC ART (1591):

  Numerical logistic is that which employs numbers; symbolic logistic that which uses symbols, as, say, the letters of the alphabet. [...] Given terms are distinguished from unknown by constant, general, and easily recognized symbols, as (say) by designating unknown magnitudes by the letter A and the other vowels E, I, O, U, and Y and given terms by the letters B, G, D and the other consonants.

- Used German $+$ and $-$, *in* for multiplication, fraction bar for division.
- Still used words/abbreviations for exponents:

$$\frac{A \text{ cubus } \textit{in } B}{C \text{ quadratum}} = \frac{A^3 B}{C^2}$$

CONSEQUENCES

- Recognized that symbols could stand not just for numbers, but anything to which arithmetic operations could be applied
- "An equation is not changed by *antithesis*" *read p. 411*
- Derived standard algebraic identities in symbols for the first time, e.g., $(A - B)(A + B) = A^2 - B^2$

- Symbolism allowed the development of a coherent theory of equations: *Two Treatises on the Recognition and Emendation of Equations*
- A quad $+$ $B2$ in A equals Z plane
- Sets $A + B = E$, so $A = E - B$ and $(E - B)^2 + 2B(E - B) = Z$, which reduces to $E^2 = Z + B^2$. Then $A = \sqrt{Z + B^2} - B$.
- He writes:

$$A \text{ is } \ell.\overline{Z \text{ plane} + B \text{ quad}} - B.$$

- First occurrence of a quadratic FORMULA

Stevin

SIMON STEVIN (1548-1620)

- Born in Belgium
- Spent his adult life in the Netherlands
- Engineer, mathematics and ballistics tutor, quartermaster general of the army
- Wrote textbooks in Dutch for subjects taught at Leiden
- Known for work on decimal fractions

- Hindu-Arabic numerals in Europe, but only for integers
- If fractions need, written off as common fractions or, in trig, sexageismals used
- Viète: if one needs to calculate $\sqrt{2}$ with a high degree of accuracy, add lots of zeros and calculate

$$\sqrt{20,000,000,000,000,000,000,000,000,000,000,000}$$

(which he did)

STEVIN'S DE THIENDE AND L'ARITHMÉTIQUE

- Aim is to show that "broken numbers" can be calculated as if one is using whole numbers
- Examples, e.g., write $0.14159 = \frac{14159}{100000}$
- Vanishing Euclidean distinction between number and magnitude, discrete and continuous.

Stevin was first to state this explicitly in *l'Arithmétique*:

- Arithmetic is the science of numbers.
- Number is that which explains the quantity of each thing:
  The root of 8 is part of its square 8. Therefore $\sqrt{8}$ is of the same matter that is 8. But the matter of 8 is number. Therefore the matter of $\sqrt{8}$ is number. And, by consequence, $\sqrt{8}$ is a number.
- From our vantage point it is hard to understand how important Stevin's contribution is, as we take it for granted. "Ultimately, he was so successful that it is difficult to understand how things were done before him."
- Process not complete until the 19th century, which wasn't that long ago!