MATH 390: CALCULUS

BEGINNINGS

Dr. Mike Janssen

Lecture 16

Question

What words, phrases, techniques, etc., come to mind when you hear the word ***calculus****? How would you define calculus to someone who's never taken a course on it? Discuss in groups of 2-3.*

5 elements quote

PLAN

1. Tangents and Extrema
2. Areas and Volumes
3. Lengths of curves

Tangents and Extrema

DIVIDING A LINE

Problem

Consider a line of length b. How can it be divided into two line segments so as to maximize the product of the lengths of the smaller segments?

FERMAT'S APPROACH

Fermat's method for maximizing a polynomial $p(x)$.

PHILOSOPHICAL QUESTION: How can one divide by $x_1 - x_2$ and then set $x_1 = x_2$?

This seemed to work in general, though Fermat noted that when $p(x)$ was complicated, division by $x_1 - x_2$ would be difficult. His solution was a process he termed ADEQUALITY.

Application to Tangents

Areas and Volumes

INFINITESIMALS AND INDIVISIBLES

- Kepler's method of INFINITESIMALS:

  in discovering laws of planetary motion, used the procedure of adding up small regions:

  > *the circumference … has as many parts as points, namely, an infinite number; each of these can be regarded as the base of an isosceles triangle with equal sides AB so that there are an infinite number of triangles in the area of the circle, all having their vertices at the center A.*

- Galileo: method of INDIVISIBLES: each geometric object is made up of objects of dimension one less

EVANGELISTA TORRICELLI (1608-1647)

- Student of Galileo
- Warned that the 'uncritical use' of indivisibles could lead to paradoxes
- Explored the volume of the solid of revolution of the hyperbola $xy = k^2$ around the y-axis from $y = a$ to $y = \infty$

FERMAT, ROBERVAL, AND AREAS UNDER PARABOLAS

- Fermat, to Gilles de Roberval, Sept. 1636: able to square "infinitely many figures composed of curved lines"
- In particular, can calculate the area of a region under any 'higher parabola' $y = px^k$
- Found that:

  > *The sum of the square numbers is always greater than the third part of the cube which has for its root the root of the greatest square, and the same sum of the squares with the greatest square removed is less than the third part of the same cube; the sum of the cubes is greater than the fourth part of the [fourth power] and with the greatest cube removed, less than the fourth part, etc.*

- That is, finding the area of the region bounded by $y = px^k$, the x-axis, and $x = x_0$ depends on:

$$\sum_{i=1}^{N-1} i^k < \frac{N^{k+1}}{k+1} < \sum_{i=1}^{N} i^k$$

- Then could see that the area is $A = \frac{x_0 y_0}{k+1}$.

$k = 2$

Lengths of Curves

JAMES GREGORY AND ISAAC BARROW

- English mathematicians
- Wrote *Universal Part of Geometry* (1668) and *Geometrical Lectures* (1670), respectively
- Material today identified as calculus
- Connected the tangent problem to the area problem via the work of van Heuraet
- Crucially, neither presented a systematic method for solving problems

Theorem

Let ZGE be any curve of which the axis is AD and let ordinates applied to this axis, AZ, PG, DE, continually increase from the initial ordinate AZ. Also let AIF be a curve such that if any straight line EDF is drawn perpendicular to AD, cutting the curves in the points E, F, and AD in D, the rectangle contained by DF and a given length R is equal to the intercepted space $ADEZ$. Also let $DE : DF = R : DT$ and join FT. Then TF will be tangent to AIF.

That is, Barrow begins with a curve $y = f(x)$ (ZGE), and constructs a new curve $AIF = g(x)$ such that $Rg(x)$ is equal to the area bounded by f between a fixed point and x. That is,

$$Rg(x) = \int_a^x f(u)du$$

.

Then: The length $t(x)$ of the subtangent to $g(x)$ is given by $Rg(x)/f(x)$, or

$$g'(x) = \frac{g(x)}{t(x)} = \frac{f(x)}{R} \text{ or } \frac{d}{dx}\int_a^x f(u)du = f(x).$$

WHO INVENTED CALCULUS?

- What is calculus?
- Ideas present in work of Archimedes
- Modern notation required to move the state of the art forward
- Fermat and Descartes (and others!) studied both sides of the FTC
- What makes calculus such a valuable tool for fulfilling the cultural mandate is not just the ideas, but the algorithms for applying them to algebraic functions