MATH 390: CALCULUS

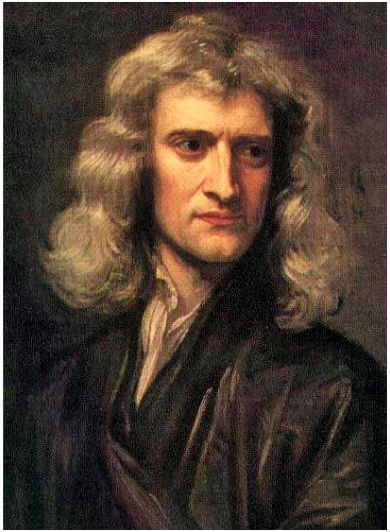
NEWTON

Dr. Mike Janssen

Lecture 17

ISAAC NEWTON (1642-1727)

- Difficult family life
- Trinity College, Cambridge in 1661
- Scholarship to study mathematics 1664
- Plague closed Cambridge 1664-1666
- Lucasian Professor of Mathematics 1669-1702
- Three works 1666-1671 developing the calculus



MAIN WORKS

- October 1666 tract on fluxions
- *On Analysis by Equations with Infinitely Many Terms*, 1669
- *A Treatise on the Methods of Series and Fluxions*, 1671
- *Principia Mathematica*, 1687

Power Series

ON POWER SERIES

Since the operations of computing in numbers and with variables are closely similar … I am amazed that it has occurred to no one (if you except N. Mercator with his quadrature of the hyperbola) to fit the doctrine recently established for decimal numbers in similar fashion to variables, especially since the way is then open to more striking consequences. For since this doctrine in species has the same relationship to Algebra that the doctrine in decimal numbers has to common Arithmetic, its operations of Addition, Subtraction, Multiplication, Division and Root extraction may easily be learnt from the latter's provided the reader be skilled in each, both Arithmetic and Algebra, and appreciate the correspondence between decimal numbers and algebraic terms continued to infinity … And just as the advantage of decimals consists in this, that when all fractions and roots have been reduced to them they take on in a certain measure the nature of integers, so it is the advantage of infinite variable-sequences that classes of more complicated terms (such as fractions whose denominators are complex quantities, the roots of complex quantities and the roots of affected equations) may be reduced to the class of simple ones: that is, to infinite series of fractions having simple numerators and denominators and without the all but insuperable encumbrances which beset the others.

Fluxions and Applications

From the *Treatise on Methods*:

1. Given the length of the space continuously [that is, at every time], to find the speed of motion at any time proposed.
2. Given the speed of motion continuously, to find the length of the space described at any time proposed.

For Newton, calculus is about MOTION, and he used FLUXIONS.

Definition

*Let x be a quantity dependent on time (called the **fluent**). Then the **fluxion** $\dot{x}$ is the speed with which x increases via its generating motion.*

To solve Problem 1:

> *Arrange the equation by which the given relation is expressed according to the dimensions of some fluent quantity, say, x, and multiply its terms by any arithmetical progression nd then by $\frac{\dot{x}}{x}$. Carry out this operation separately for each one of the fluent quantities and then put the sum of all the products equal to nothing, and you have the desired equation.*

EXAMPLE

Calculate the fluxion relationship given $x^3 - ax^2 + axy - y^3 = 0$.

JUSTIFICATION

Via INFINITESIMALS: The MOMENT of a fluent quantity is the amount by which it increases in an "infinitely small" period of time. The increase of x over an infinitely small time o is $x + \dot{x}o$.

APPLICATIONS OF FLUXIONS

PROBLEM: Find maximum/minimum: it is then that fluxions are 0, i.e., $\dot{x} = 0$.

EXAMPLE.

PROBLEM: Find tangents.

Fluents and Areas

On Problem 2:

> *Since this problem is the converse of the preceding, it ought to be resolved the contrary way: namely by arranging the terms multiplied by $\dot{x}$ according to the dimensions of x and dividing by $\dot{x}/x$ and then by the number of dimensions, … by carrying out the same operation in the terms multiplied by … $\dot{y}$, and, with redundant terms rejected, setting the total of the resulting terms equal to nothing.*

Newton developed a table of integrals for algebraic functions; no transcendental functions such as trig, logarithms, etc.

LATER LIFE

- Once the calculus was in hand, Newton was able to recast physics (mechanics) in terms of forces in the *Principia*
- Note that physics is often presented as the reason for Newton's work on calculus, but evidence suggests this is not the case
- *Principia* revolutionized the next 200 years of physics
- The calculus was a footnote as it was not widely published, and then not until decades after their initial discovery
- Newton became Master of the Royal Mint in 1696 and president of the Royal Society in 1703.