MATH 390: CALCULUS

LEIBNIZ

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Lecture 18

GOTTFRIED WILHELM LEIBNIZ (1646-1716)

- Son of the vice chairman of the philosophy faculty at Leipzig
- Major contributions to 17th century rationalism
- Introduced to advanced mathematics in 1663; hoped to mathematize all of human thought via appropriate notation
- Studied Descartes' *Geometry* and Pascal

Let A, B, C, D, E be an increasing sequence of numbers, and L, M, N, P the sequence of differences.

Then $E - A = L + M + N + P$.

LEIBNIZ'S NOTATION

- d: first letter of the Latin *differentia*
- $\int$: elongated S, the first letter of the latin *summa*
- dy and $\sum y$ are variables; dy is an actual difference, and $\sum y$ is an actual sum
- That is, d and $\int$ are operators
- Since dy is a variable, it can be operated on again: $d\,dy = d^2y$

The Calculus of Differentials

Pascal's Differential Triangle

TRANSMUTATION THEOREM

$$\int_0^{x_0} y\,dx = \frac{1}{2}\left(x_0 y_0 + \int_0^{x_0}\left(y - x\frac{dy}{dx}\right)\,dx\right)$$

In 1675:

- If a is constant, $da = 0$
- $d(v \pm y) = dv \pm dy$
- "Let us now investigate whether $dx\, dy$ is the same thing as $d(xy)$, and whether dx/dy is the same thing as $d(x/y)$."
- $d(v/y) = (\pm v dy \mp y dv)/y^2$

In 1684 paper:

- $d(x^n) = nx^{n-1}$
- $d\sqrt[b]{x^a} = (a/b)\sqrt[b]{x^{a-b}}dx$

APPLICATION TO EXTREMA

- dv is positive when v is increasing and negative when v is decreasing, since the ratio dv/dx gives the slope of the tangent line (and dx is positive)
- $dv = 0$ when v is neither increasing or decreasing; this is then a maximum or a minimum, depending on how v was moving to the left and right
- "Where the increment is maximum or minimum, or where the increments from decreasing turn into increasing, or the opposite, there is a POINT OF INFLECTION", i.e., when $d\,dv = 0$

Given Leibniz's observation that sums and differences are inverse processes, the FTC was obvious.

"I represent the area of a figure by the sum of all the rectangles contained by the ordinates and the differences of the abscissae,", i.e,. as $\int y\, dx$.

> *I obtain the area of a figure by finding the figure of its summatrix or quadratrix; and of this indeed the ordinates are to the ordinates of the given figure in the ratio of sums to differences.*

That is, to find the area under the curve with ordinates y, find a curve z such that $y = dz$. Then

$$\int y\, dx = z.$$

But Leibniz was more interested in solving differential equations than finding areas.

Newton, Leibniz, and the Spread of Calculus

CORRESPONDENCE

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PRIORITY

Who invented calculus?

- Both discovered their calculus at essentially the same time
- Approaches were entirely different: Newton via velocity and distance and Leibniz via sums and differences
- Newton's work not published until early 1700s despite being well-known in England
- Leibniz in late 1600s and he and the Bernoullis were successful in applying it
- Accusation of plagiarism by the English
- Accusation of plagiarism by the Bernoullis
- Royal Society (of which Newton was president) appointed a commission to look into the charges. Leibniz was found guilty.
- Resulted in a cessation of mathematical collaboration between England the continent, to the detriment of England

NEW CALCULUS TEXTS

- L'Hospital (1696): *Analysis of Infinitely Small Quantities for the Understanding of Curves*
- Learned it from Johann Bernoulli
- Ditton (1706): *An Institution of Fluxions*
- Hayes (1704): *A Treatise of Fluxions*