MATH 390: ANALYSIS IN THE EIGHTEENTH CENTURY

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Lecture 19

THE BERNOULLIS

Jakob Bernoulli (1654-1705)

Johann Bernoulli (1667-1748)

- Johann posed, with Leibniz, lots of problems involving differential equations
- Developed solution techniques
- Leibniz: separation of variables, solutions to first-order equations like

$$\frac{dy}{dx} + ny = -m$$

THE BRACHISTOCHRONE PROBLEM

PROBLEM

If two points A and B are given in a vertical plane, to assign to a mobile particle M the path AMB along which, descending under its own weight, it passes from the point A to the point B in the briefest time.

Posed the problem in ACTA ERUDITORUM

DIFFERENTIAL EQUATIONS AND TRIG FUNCTIONS

- Early eighteenth century solution of

$$dt = \frac{c\,ds}{\sqrt{c^2 - s^2}}$$

  would be to solve for $t = c \arcsin(s/c)$ by considering the geometry.
- Sine was not considered a function in the modern sense, though the exponential and logarithmic functions were
- In 1739, Leonhard Euler realized that a sine function would enable closed-form solutions to higher-order differential equations to be given

LEONHARD EULER (1707-1783)

- Swiss mathematician
- Spent much time in Russia
- Called the most prolific mathematician of all time
- Influenced all areas of math


OEUVRES DE DESCARTES III
THE ELEMENTS
THE ELEMENTS

Logarithms of Negative and Complex Numbers

THE QUESTION

- What is the logarithm of a negative number?
- Johann Bernoulli: $\ln(-x) = \ln(x)$, since the rectangular hyperbolas used to define the logarithm had two branches
- Leibniz: logarithms of negative numbers were “impossible”
- In a letter to Euler, Bernoulli argued:

$$d\ln(-x) = \frac{-1}{-x}dx = \frac{1}{x}dx = d\ln(x)$$

- Euler’s response was to use some of Bernoulli’s own work to argue that $\ln(-1) = \pi i$.

Calculus Texts

IN THE VERNACULAR

- Thomas Simpson's *Treatise of Fluxions* (1737)
- Colin Maclaurin's *A Treatise of Fluxions* (1742)
- Maria Agnesi's *Foundations of Analysis for the Use of Italian Youth* (1748)

LATIN TEXTS

Euler's trilogy:

- *Introduction to Analysis of the Infinite* (1748)
- *Methods of the Differential Calculus*
- *Methods of the Integral Calculus*

FRENCH REVOLUTION

- No more aristocracy in France
- Weakened elsewhere
- Need for educating a new class of students entering the sciences
- Development of rigorous foundation for the calculus follows

The Foundations of Calculus

A NEED FOR RIGOR

- Euclid’s *Elements* the gold standard of rigor
- Newton, Leibniz, Euler knew they were correct and didn’t worry much about “rigorous” proof
- Yet their explanations left something to be desired
- What are fluxions? Differentials?

GEORGE BERKELEY'S (1685-1753) *THE ANALYST*

- Work addressed to a "infidel mathematician" supposed to be Edmond Halley
- Did not deny the usefulness of the calculus, but wanted to demonstrate that no one had any good reasons or explanations for the results they were using
- Took issue with the use of moments and fluxions: how can one calculate with a nonzero quantity, then set it equal to zero, and retain a consequence of that calculation?
- Quotes p. 629

MACLAURIN'S RESPONSE

- Newton and Leibniz were unable to defend the calculus, but Colin Maclaurin did
- *Treatise of Fluxions*: goal to "deduce those Elements [of the theory of Fluxions] after the Manner of the Ancients"
- Does not use limits
- Shows that "infinitesimals" in Newton's arguments can always be replaced by finite quantities
- Largely successful in his goal, but at the cost of influence, calculation speed, and readability

EULER AND D'ALEMBERT

- Continental effort to firm up the foundations of calculus
- Euler developed the idea that the ratios involved in derivatives WERE $0 : 0$.
- Therefore pushed back on the Berkeleys: "nothing is neglected except that which is actually nothing"
- d'Alembert introduced the limit

LAGRANGE AND POWER SERIES

- Joseph-Louis Lagrange (1736-1713)
- Gave a precise definition of the derivative by eliminating all reference to infinitesimals, fluxions, zeros, and even limits
- Initial ideas in 1772, developed fully in 1797: *The Theory of Analytic Functions, containing the principles of the differential calculus, released from every consideration of the infinitely small or the evanescent, of limits or of fluxions, and reduced to the algebraic analysis of finite quantities*
- Example
- Lagrange developed derivatives, FTC (p. 635), etc. using power series. The notion of limit is implicit in his FTC, however
- Yet used many assertions, particularly about the existence of power series for every function, that he did not prove