MATH 390: POST-EUCLIDEAN GEOMETRY

Dr. Mike Janssen
Lecture 23

THE STATE OF THE PARALLEL POSTULATE

- In the eighteenth century, Saccheri and Lambert attempted to prove Euclid's fifth postulate by assuming it false and deriving a contradiction
- Saccheri thought he was successful
- Lambert knew he wasn't
- Gauss privately worked out much of what we're about to talk about, but never published anything
- However, as Saccheri and Lambert were unsuccessful, other mathematicians began to believe that a plane geometry in which the hypothesis of the acute angle was true could exist.

NIKOLAI IVANOVICH LOBACHEVSKY (1792-1856)

- 1826 lecture at Kazan University
- Described a geometry having more than one parallel to a given line through a given point
- 1829: *On the Principles of Geometry*

JÁNOS BOLYAI (1802-1860)

- 1831: appendix to a geometric work of his father: "Appendix Exhibiting the Absolutely True Science of Space, Independent of the Truth or Falsity of Axiom XI of Euclid (that can never be decided a priori)"
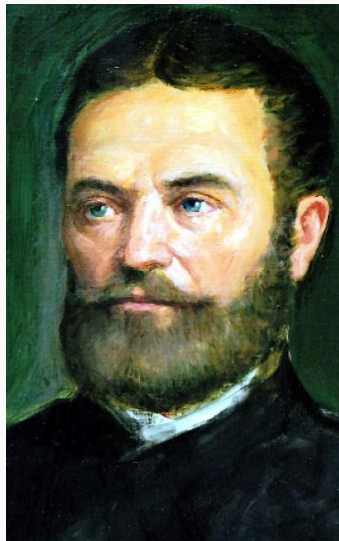
LOBACHEVSKY'S APPROACH

- Begin with a summary of results independent of the parallel postulate
- New definition for parallels.

Definition

All straight lines which in a plane go out from a point can, with reference to a given straight line in the same plane, be divided into two classes–cutting and not-cutting. The boundary lines of the one and other class of those lines will be called parallel to the given line.

LOBACHEVSKY'S CONCLUSIONS

- This is "a sufficient foundation for considering the assumption of [non-Euclidean] geometry as possible."

- Gauss also realized the creation of a new, apparently valid geometry in which the parallel postulate did not hold meant that there was no "necessity" to Euclid's work

- That is, one cannot conclude that Euclidean geometry is automatically the "correct" version of geometry in the world in which we live.

- Bolyai derived most of the same results as Lobachevsky, but included more results in absolute (neutral) geometry

- Results mostly ignored for a few decades

BERNHARD RIEMANN (1826-1866)

- Son of a Protestant minister
- Began studying to be a pastor
- Went to Göttingen to study theology, but started studying mathematics under Gauss
- Built on Gauss' ideas to create the notion of a **manifold**
- (Smooth) Manifolds look "locally" like Euclidean space
- Not published until after Riemann's death

PROJECTIVE GEOMETRY

- Parallel lines appear to meet
- Jean-Victor Poncelet (1788-1867): *Projective Properties of Figures*
- Points at infinity necessitated a coordinate system, introduced by Julius Plücker in 1831
- Duality of point and line: $ax + by + ct = 0$
- Fano plane (Gino Fano, 1871-1952)

GRAPH THEORY

- First defined by Camille Jordan in 1869 (an **assemblage of lines**)
- WR Hamilton's *Icosian* game: find a Hamilton path
- Arthur Cayley (1857) introduced the notion of a **tree**

THE FOUR-COLOR PROBLEM

- De Morgan, in an 1852 letter to Hamilton:
  > *A student of mine [Frederick Guthrie] asked me today to give him a reason for a fact which I did not know was a fact—and do not yet. He says that if a figure be anyhow divided and the compartments differently coloured so that figures with any portion of common boundary line are differently coloured—four colours may be wanted, but not more...My pupil says he guessed it in colouring a map of England. The more I think of it, the more evident it seems.*

- Hamilton was uninterested, but Cayley kept searching for a proof.

- Six Color Theorem