MATH 390: MAINSTREAM PHILOSOPHIES OF MATHEMATICS

FORMALISM

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Lecture 26

OVERVIEW

- A casual interaction with math reveals that lots of (modern) mathematical activity seems to be about manipulating linguistic symbols according to certain rules
- *Formalism* is the claim that math, in its essence, is the manipulation of characters
- In this case, math need not be about *anything*
- Mathematicians have had occasion to introduce symbols which, at the time, had no clear interpretation
- Multiple versions of formalism; these perspectives are more popular among mathematicians than philosophers of mathematics

TERM FORMALISM

- Mathematics is about characters or symbols
- That is, the entities of mathematics are identified with their name. The complex number $8 + 2i$ is just the symbol '$8 + 2i$'
- The subject matter of mathematics is thus the symbols of mathematics, and propositions are either true or false
- Mathematical knowledge is then knowledge of how the characters/symbols are related to one another, and the rules by which they may be manipulated
- Some term formalists may distinguish between *types* and *tokens*

FREGE'S CRITIQUE

Consider the equation

$$5 + 7 = 6 + 6.$$

What can this mean from a formalist perspective?

GAME FORMALISM

- Mathematics is a game played with linguistic characters
- Radical versions: symbols are meaningless, and formulas/propositions do not express truth or falsity about any subject-matter.
- There is no more meaning to mathematics than there is to the pieces and rules of chess
- Moderate versions: maybe there is meaning, but if so, it's irrelevant

Despite Frege's critique, his goal of developing a *formal system* fed a version of formalism.

CRITIQUES

- If math has no meaning, why is it useful for science?
  An arithmetic without thought as its content will also be without possibility of application. Why can no application be made of a configuration of chess pieces? –Frege (1903)
- Formalist reply: applications are not a part of mathematics, so we don't need to account for them

DEDUCTIVISM

- Suppose someone provides an interpretation of the axioms of arithmetic so that they are true
- Why should any theorems deduced from them also be true?
- How do we know that the rules of the 'arithmetic-game' take us from truths to truths?
- A *deductivist* accepts that the rules of inference (logic) preserve truth, but insists that the axioms have no set meaning. They can be treated as arbitrary, or given an interpretation at will
- One then distinguishes logical terms ('and', 'or', etc) from mathematical ('point', 'number', etc)
- Pairs well with developments in mathematics in the 19th/20th centuries

HILBERT'S SECOND PROBLEM

- 'When we are engaged in investigating the foundations of a science, we must set up a system of axioms which contains an exact and complete description of the relations subsisting between the elementary ideas of that science. The axioms set up are at the same time the definitions of those elementary ideas...'
- Meta-mathematics: prove theorems about formal systems
- Hilbert believed that if a collection of axioms was consistent, then they are true and the things of which they speak exist.
- In the milieu of the turn of the 20th century, Hilbert wanted to 'establish once and for all the certitude of mathematical methods' by finding consistent axioms for mathematics.

FINITISM

- Finitary arithmetic: statements about natural numbers that are *effectively decidable*–there is an algorithm that can determine if they are true in finite time
- Finitary arithmetic is about natural numbers, which should be identified with symbols, e.g.,

$$|, ||, |||, \ldots$$

- Then we should not be able to derive a statement Φ in T unless it can be derived using finitary mathematics

INCOMPLETENESS

Let T be a formal deductive system that contains a certain amount of arithmetic. Assume that the syntax of T is *effective* in the sense that we can determine the truth-value of a statement in T in finite time.

- Gödel showed that there is a sentence G such that, if T is ω-consistent, neither G nor $\neg G$ is a theorem of T.
- He also showed that if T is consistent, we cannot derive the consistency of T within T.
- This seems to have dealt a blow to the existence of any unifying formalization of all of arithmetic (and thus all of mathematics)

DISCUSSION QUESTIONS

- What meaning is found in mathematics?
- Does mathematics ever *feel* like a formal game?
- What might it look like for a formalist to teach mathematics?
- What do we make of Gödel’s Theorems?
- How might a neo-Kuyperian respond to the formalist perspective?