MATH 390: TWENTIETH CENTURY MATHEMATICS

STATISTICS AND COMPUTATION

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Lecture 27

Statistics

RONALD FISHER (1890-1962)

- Chief statistician at British agricultural experiment station
- Eugenicist
- *Statistical Methods for Research Workers* (1925)
- *The Design of Experiments* (1935)

FISHER'S METHOD FOR INFERENCE

1. Null hypothesis: a statement that two situations are no different
2. Observations: what are we observing, and how can we describe their deviation from the null hypothesis? More art than science.
3. Measure: by how much does our observed deviation differ from what is expected under the null hypothesis? Usually a probability called a p-value.

We then reject the null hypothesis if the p-value is less than some predetermined amount, usually 0.05 or 0.01, though p-values must be handled with care.

LADY TASTING TEA

- A woman of Fisher's acquaintance claimed to be able, when tea is mixed with milk, to tell whether the tea was poured first or the milk.
- Experiment: the woman is given 8 cups of tea, four prepared one way and four the other. Her job is to group the cups by preparation method.
- There are $\binom{8}{4} = 70$ ways of selecting a set of four out of a set of eight. If she couldn't tell the difference, she would have a $1/70 = 0.01428571$ chance of selecting correctly.
- The chances of her getting three right without being able to tell the difference are $16/70 \approx 0.228$, which is high enough that we wouldn't reject the null.
- Fisher notes that other experimental designs are possible (more right than wrong, etc)

ON REJECTING THE NULL

- Fisher notes that we do not disprove the null hypothesis
- Also that rejecting the null does not prove the alternative, and we should not draw conclusions from nonsignificant results
- And that we should not necessarily dismiss the null out of hand on the basis of one experiment, no matter how well designed or how strong the evidence

WWII AND AFTER

- Need for better statistical procedures driven first by the New Deal, then by World War II
- Statistical Research Group: Columbia University
- Abraham Wald and survivorship bias
- By the 1950s, it was clear that statistical computations would be done on computers, thus enabling the exploration of large datasets

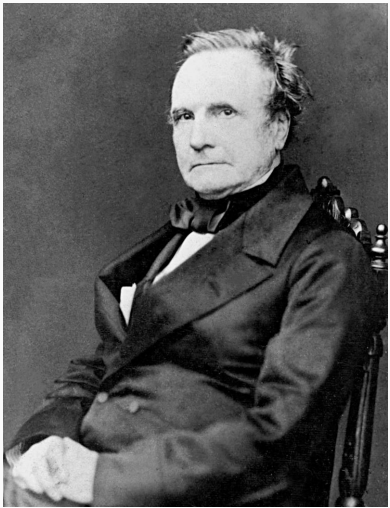
Computation

PREHISTORY OF COMPUTERS

- "Computer" was a type of job, possibly as far back as Ptolemy's *Almagest*
- Pascal built a mechanical adding/subtracting machine
- Leibniz built a machine that did multiplication and division

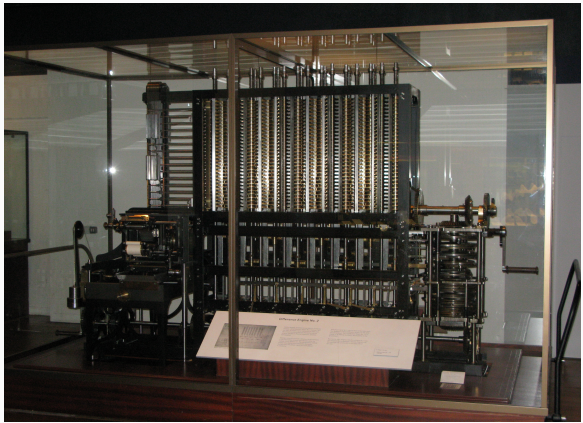
CHARLES BABBAGE (1791-1871)

- English mathematician and polymath
- Had the idea to use the steam engine to drive a calculating machine
- First machine: the *difference engine*
- Worked with Ada Byron King Lovelace on the *analytical engine*



DIFFERENCE ENGINE (1820'S)

- Noted: the nth-order differences for a degree n polynomial $f(x)$ are always constant as f is evaluated at an arithmetic progress such as $1, 2, 3, \ldots$.
- Approximations by polynomials (e.g., Taylor polynomials)
- Never completed

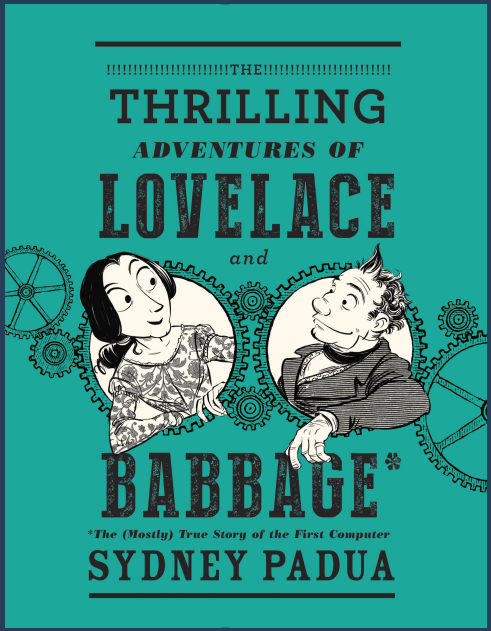


THE ANALYTICAL ENGINE

- Two parts: the *store* and *mill*
- Babbage borrowed an idea from the weaving industry: punched cards
- Babbage made detailed descriptions (e.g., 300 sheets of engineering drawings, thousands of pages of notes), but never constructed it
- Based on his notes, it is thought that the technology was sufficient, but the resources lacking
- 1840: Seminars on the analytical engine in Italy
- Italian scientist summarized in an article translated to English in 1843, supplemented by 40 pages of notes by Ada Lovelace

LOVELACE'S NOTES

- Described not just the functioning, but explicitly how the engine would solve various problems
- In short, she wrote the first *computer program* for calculating Bernoulli numbers
- Includes loops, decision steps, etc.


THE
THRILLING
ADVENTURES OF
LOVELACE
and
BABBAGE*
*The (Mostly) True Story of the First Computer
SYDNEY PADUA

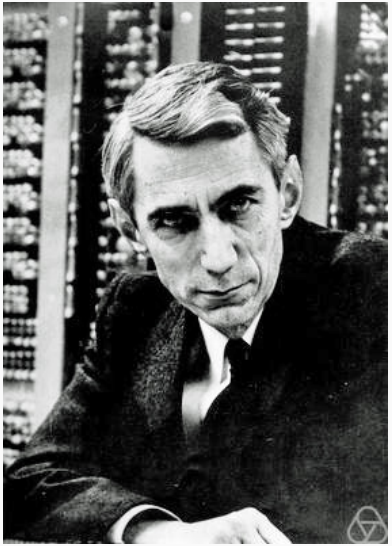

ADA was the daughter of "mad, bad, and dangerous to know" poet and nutcase Lord Byron.
Her mother Anabel fled ~~the exploding planet~~ her husband but was afraid their daughter would inherit his WILD BLOOD!!
ADA MUST NEVER BECOME POETICAL!
ONLY ONE THING HAS THE POWER TO SUBDUE POETRY...
...MATHEMATICS!!
ADVANCED CALCULUS

ALAN TURING (1912-1954) AND COMPUTABILITY

- Desired an answer to the question: what is a computation, and can we know if a given computation can be carried out?
- Abstracted the essential parts of computation and formulated these in terms of a theoretical machine now called a *Turing machine* (1936)
- Abstract machine consisting of:
  - a finite set of states/configurations
  - a finite set of symbols to be read/written by the machine
  - a process of changing the states and symbols to be read
- Machine is supplied with an infinite tape running through it, which can move forward or backward (memory)
- Turing showed that his machine was capable of arbitrary computations
- Led to the all-purpose computers we know today

CLAUDE SHANNON (1916-2001)

- MIT Master's thesis: applied Boolean logic to construction of switching circuits
- "Father of information theory"



JOHN VON NEUMANN (1903-1957)

- Hungarian-American mathematician
- Institute for Advanced Study
- Four sections: an arithmetic unit, a memory, a control, and an input-output device
- Finished in 1951 and basis for everything after